# Physics in Nuclear Medicine

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A Subsidiary of Harcourt Brace Jovanovich, Publishers New York London Paris San Diego San Francisco São Paulo Sydney Tokyo Toronto

## **Basic Atomic and Nuclear Physics**

Radioactivity is a process involving events in individual atoms and nuclei. Before discussing radioactivity, therefore, it is worthwhile to review some of the basic concepts of atomic and nuclear physics.

#### A. MASS AND ENERGY UNITS

Events occurring on the atomic scale, such as radioactive decay, involve masses and energies much smaller than those encountered in the events of our everyday experiences. Therefore they are described in terms of mass and energy units more appropriate to the atomic scale.

The basic unit of mass is the *universal mass unit*, abbreviated u. One u is defined as being equal to exactly  $\frac{1}{12}$  the mass of a  $\frac{12}{12}$ C atom. † A slightly different unit, commonly used in chemistry, is the *atomic mass unit* (amu), based on the average weight of oxygen isotopes in their natural abundance. In this text, except where indicated, masses will be expressed in universal mass units, u.

The basic unit of energy is the *electron volt*, abbreviated eV. One eV is defined as the amount of energy acquired by an electron when it is accelerated through an electrical potential of one volt. Basic multiples are the keV (kilo electron volt; 1 keV = 1000 eV) and the MeV (kega electron volt; 1 MeV = 1000 keV = 1,000,000 eV).

Mass m and energy E are related to each other by Einstein's equation  $E = mc^2$ , where c is the velocity of light. According to this equation, 1 u of mass is equivalent to 931.5 MeV of energy.

Table 1-1
Mass and Energy Units

Multiply → To Obtain	Ву	To Obtain ← Divide		
u	$1.66043 \times 10^{-24}$	g		
u	$4.86 \times 10^{-26}$	. <b>OZ</b>		
u	1.00083	amu		
eV	$1.6021 \times 10^{-12}$	ergs		
eV	$3.83 \times 10^{-20}$	g∙cal		
u	931.478	MeV		

Relationships between various units of mass and energy are summarized in Table 1–1. Universal mass units and electron volts are very small, yet, as we shall see, they are quite appropriate to the atomic scale.

#### **B. ELECTROMAGNETIC RADIATION (PHOTONS)**

Atomic and nuclear processes often result in the emission of electromagnetic radiation, such as x rays (Section C.3), and  $\gamma$  rays (gamma rays, Section D.5). Electromagnetic radiation consists of oscillating electrical and magnetic fields traveling through space with the velocity of light, c (approximately 3  $\times$  10<sup>8</sup> meters/sec in vacuum). The wavelength  $\lambda$  and frequency  $\nu$  of these oscillating fields are related by

$$\lambda \nu = c \tag{1-1}$$

In some cases, e.g., when interacting with atoms, electromagnetic radiations behave as discrete "packets" of energy, called *photons* (also called *quanta*). A photon is a packet of electromagnetic energy having no mass or electrical charge that also travels through space at the velocity of light. The energy of a photon, E, and the wavelength  $\lambda$  of its associated electromagnetic field are related by

$$E(keV) = 12.4/\lambda(A)$$
 (1-2)

$$\lambda(\text{Å}) = 12.4/\text{E}(\text{keV}) \tag{1-3}$$

Table 1–2 lists approximate photon energies and wavelengths for different parts of the electromagnetic spectrum. Note that x and  $\gamma$  rays occupy the highest-energy, shortest-wavelength end of the spectrum; x- and  $\gamma$ -ray photons have energies in the keV–MeV range, whereas visible light photons, for example, have energies of only a few eV. As a consequence of their short wavelength

Table 1–2						
Approximate Photon Energy and	l Wavelength	Ranges	for	Different	Types	of
Electromagnetic Radiation						

Туре	Energy (eV)	Wavelength	
Radio, TV, radar	10 <sup>-9</sup> -10 <sup>-3</sup>	10 <sup>-1</sup> –10 <sup>5</sup> cm	
Infrared	$10^{-3}-2$	$10^{-4}$ – $10^{-1}$ cm	
Visible	2–3	4000–7000 ņ	
Ultraviolet	3–25	50–4000 Å	
x rays ~	25-10 <sup>5</sup>	$10^{-1}$ –50 Å	
γ rays	$10^4 - 10^6$	$10^{-2}$ –1 Å	

 $<sup>^{\</sup>dagger}1 \text{ Å} = 10^{-8} \text{ cm}.$ 

and high energy, x and  $\gamma$  rays interact with matter quite differently from other, more familiar types of electromagnetic radiation. These interactions are discussed in detail in Chapter 9.

#### C. ATOMS

#### 1. Composition and Structure

All matter is comprised of atoms. An atom is the smallest unit into which a chemical element can be broken down without losing its chemical identity. Atoms combine to form molecules and chemical compounds, which in turn combine to form larger, macroscopic structures.

The existence of atoms was first postulated on philosophical grounds by Ionian scholars in the 5th century B.C. The concept was formalized into scientific theory early in the 19th century, owing largely to the work of the chemist Dalton and his contemporaries. The exact structure of atoms was not known, but at that time they were believed to be indivisible. Later in the century (1869), Mendeleev produced the first *periodic table*, an ordering of the chemical elements according to the weights of their atoms and arrangement in a grid according to their chemical properties. For a time it was believed that completion of the periodic table would represent the final step in understanding the structure of matter.

Events of the late 19th and early 20th centuries, beginning with the discovery of x rays by Roentgen (1895) and radioactivity by Bequerel (1896), revealed that atoms had a substructure of their own. In 1910, Rutherford presented experimental evidence indicating that atoms consisted of a massive, compact, positively charged core, or *nucleus*, surrounded by a diffuse cloud of relatively light, negatively charged *electrons*. This model came to be known as the *nuclear atom*. The number of positive charges in the nucleus is called the *atomic number* of the nucleus (Z). In the electrically neutral atom, the number of orbital electrons is sufficient to balance exactly the number of positive

charges, Z, in the nucleus. The chemical properties of an atom are determined by orbital electrons; therefore the atomic number Z determines the *chemical element* to which the atom belongs. A listing of chemical elements and their atomic numbers is given in Appendix A.

According to classical theory, orbiting electrons should slowly lose energy and spiral into the nucleus, resulting in atomic "collapse." This is obviously not what happens. The simple nuclear model therefore needed further refinement. This was provided by Niels Bohr in 1913, who presented a model that has come to be known as the *Bohr atom*. In the Bohr atom there is a set of stable electron orbits or "shells" in which electrons can exist indefinitely without loss of energy. The diameters of these shells are determined by *quantum numbers*, which can have only integer values (n = 1, 2, 3, ...). The innermost shell (n = 1) is called the K shell, the next the L shell (n = 2), followed by the M shell (n = 3), N shell (n = 4), and so forth.

Each shell is actually comprised of a set of orbits, called substates, which differ slightly from one another. Each shell has 2n-1 substates, where n is the quantum number of the shell. Thus the K shell has only one substate; the L shell has three substates, labeled  $L_{\rm I}$ ,  $L_{\rm III}$ , and so forth. Figure 1-1 is a schematic representation of the K and L shells of an atom. The M shell and other higher shells have larger diameters.

The Bohr model of the atom was further refined with the statement of the *Pauli Exclusion Principle* in 1925. According to this principle, no two orbital electrons in an atom can move with exactly the same motion. Because of different possible electron "spin" orientations, more than one electron can exist in each substate (Figure 1-1); however, the number of electrons that can exist in any one shell or its substates is limited. For a shell with quantum number n,

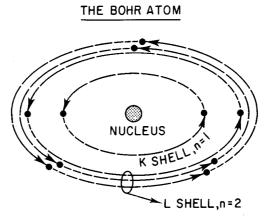


Fig. 1-1. Schematic representation of the Bohr model of the atom; *n* is the quantum number of the shell. The K shell has one substate and the L shell has three substates. Each substate has two electrons.

the maximum number of electrons allowed is  $2n^2$ . Thus the K shell (n = 1) is limited to two electrons, the L shell (n = 2) to eight electrons, and so forth.

The Bohr model is actually an oversimplification. According to modern theories the orbital electrons do not move in precise circular orbits, but rather in imprecisely defined "regions of space" around the nucleus, sometimes actually passing through the nucleus; however, the Bohr model is quite adequate for the purposes of this text.

## 2. Electron Binding Energies and Energy Levels

In the most stable configuration, orbital electrons occupy the innermost shells of an atom, where they are most "tightly bound" to the nucleus. For example, in carbon, which has six electrons, two electrons (the maximum number allowed) occupy the K shell, and the four remaining electrons are found in the L shell. Electrons can be moved to higher shells or completely removed from the atom, but doing so requires an energy input to overcome the forces of attraction that "bind" the electron to the nucleus. The energy may be provided, for example, by a particle or a photon striking the atom.

The amount of energy required to completely remove an electron from a given shell in an atom is called the binding energy of that shell. It is symbolized by the notation  $K_B$  for the K shell, †  $L_B$  for the L shell ( $L_{IB}$ ,  $L_{IIB}$ ,  $L_{IIIB}$  for the L shell substates), and so forth. Binding energy is greatest for the innermost shell; i.e.,  $K_B > L_B > M_B$ . . . . Binding energy also increases with the positive charge (atomic number Z) of the nucleus, since a greater positive charge exerts a greater force of attraction on an electron. Therefore binding energies are greatest for the heaviest elements. Values of K shell binding energies for the elements are listed in Appendix A.

The energy required to move an electron from an inner to an outer shell is exactly equal to the difference in binding energies between the two shells. Thus the energy required to move an electron from the K shell to the L shell in an atom is  $K_B - L_B$  (with slight differences for different L shell substates).

Binding energies and energy differences are sometimes displayed on an energy level diagram. Figure 1-2 shows such a diagram for the K and L shells of the element iodine. The top line represents an electron completely separated from the parent atom ("unbound" or "free" electron). The bottom line represents the most tightly bound electrons, i.e., the K shell. Above this are lines representing substates of the L shell. (The M shell and other outer shell lines would be just above the L shell lines.) The distance from the K shell to the top level represents the K shell binding energy for iodine, i.e., 33.2 keV. To move a K shell electron to the L shell requires about 33 - 5 = 28 keV of energy.

<sup>†</sup>Sometimes the notation  $K_{ab}$  is also used.

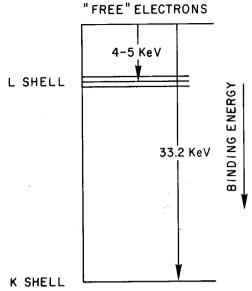


Fig. 1-2. Electron energy level diagram for an iodine atom. Vertical axis represents energy required to remove orbital electrons from different shells (binding energy). Removing an electron from the atom, or going from an inner (e.g., K) to an outer (e.g., L) shell, requires an energy input, whereas an electron moving from an outer to an inner shell results in an emission of energy from the atom.

#### 3. Atomic Emissions

When an electron is removed from one of the inner shells of an atom, an electron from an outer shell promptly moves in to fill the vacancy, and energy is released in the process. The energy released when an electron drops from an outer to an inner shell is exactly equal to the difference in binding energies between the two shells. The energy may appear as a photon of electromagnetic radiation (Figure 1–3). Electron binding energy differences have exact characteristic values for different elements; therefore the photon emissions are called characteristic radiation or characteristic x rays. The notation used to identify characteristic x rays from various electron transitions is summarized in Table 1–3. Note that some transitions are not allowed.

As an alternative to characteristic x-ray emission, the atom may undergo a process known as the *Auger* (pronounced oh-zhay') *effect*. In the Auger effect, an electron from an outer shell again fills the vacancy, but the energy released in the process is transferred to another orbital electron. This electron is then emitted from the atom instead of characteristic radiation. The process is shown schematically in Figure 1–4. The emitted electron is called an *Auger electron*.

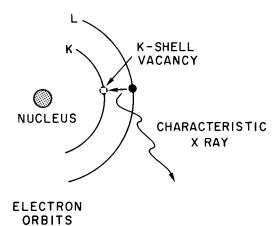


Fig. 1-3. Emission of characteristic x rays occurs when orbital electrons move from an outer shell to fill an inner-shell vacancy. ( $K_{\alpha}$  x-ray emission illustrated.)

The kinetic energy of an Auger electron is equal to the difference between the binding energy of the shell containing the original vacancy and the sum of the binding energies of the two shells having vacancies at the end. Thus the kinetic energy of the Auger electron emitted in Figure 1–4 is  $K_B - 2L_B$  (ignoring small differences in L-substate energies).

Two orbital vacancies exist after the Auger effect occurs. These are filled by electrons from other outer shells, resulting in the emission of additional characteristic x rays or Auger electrons.

Whether a particular vacancy will result in emission of a characteristic x ray or an Auger electron is a matter of probabilities. The probability that it will yield characteristic x rays is called the *fluorescent yield*, symbolized by  $\omega_K$  for

Table 1-3
Some Notation Used for Characteristic X Rays

Shell With Vacancy	Shell From Which Filled	Notation
K	$L_{r}$	Not allowed
K	$\mathbf{L}_{\mathbf{n}}$ .	$K_{\alpha 2}$
K	$L_{m}^{"}$	$K_{\alpha 1}^{\alpha 2}$
K	$\mathbf{M}_{t}^{m}$	Not allowed
K	$\mathbf{M}_{_{\mathbf{II}}}^{^{\mathbf{I}}}$	$K_{\beta 3}$
K	$\mathbf{M}_{_{\mathbf{III}}}^{^{\mathbf{II}}}$	$K_{\beta 1}$
K	$N_{r}$	Not allowed
K	$N_{\Pi,\Pi}$	$K_{\beta 2}$
$L_{\rm n}$	$\mathbf{M}_{i\mathbf{v}}$	$L_{\beta i}^{\mu z}$
$L_{m}^{"}$	$M_{iv}$	$L_{\alpha 2}^{p_1}$
$\mathbf{L}_{\mathbf{m}}^{\mathbf{m}}$	$M_{v}^{N}$	$L_{\alpha 1}^{\alpha 2}$

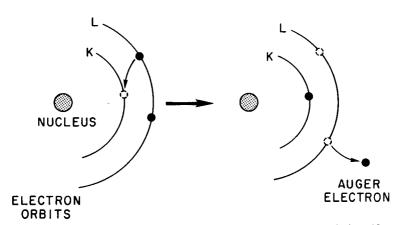


Fig. 1-4. Emission of an Auger electron as an alternative to x-ray emission. No x ray is emitted.

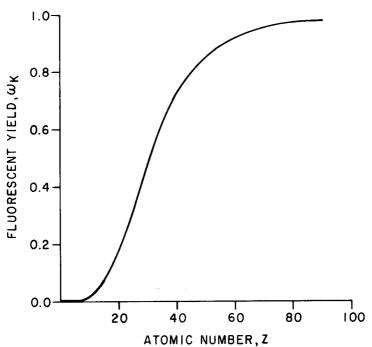


Fig. 1-5. Fluorescent yield  $\omega_{\rm K}$ , or probability that an orbital electron shell vacancy will yield characteristic x rays rather than Auger electrons, versus atomic number Z of the atom.

the K shell,  $\omega_L$  for the L shell, and so forth. Figure 1-5 is a graph of  $\omega_K$  versus Z. Both characteristic x rays and Auger electrons are emitted by all elements, but heavy elements are more likely to emit x rays (large  $\omega$ ), whereas light elements are more likely to emit electrons (small  $\omega$ ).

The notation used to identify the shells involved in Auger electron emission is  $e_{abc}$ , where a identifies the shell with the original vacancy, b the shell from which the electron dropped to fill the vacancy, and c the shell from which the Auger electron was emitted. Thus the electron emitted in Figure 1–4 is a KLL Auger electron, symbolized by  $e_{KLL}$ . In the notation  $e_{Kxx}$ , the symbol x is arbitrary, referring to all Auger electrons produced from initial K shell vacancies.

#### D. THE NUCLEUS

#### 1. Composition

The atomic nucleus is comprised of *protons* and *neutrons*. Collectively these particles are known as *nucleons*. The properties of nucleons and electrons are summarized in Table 1–4.

Nucleons are much more massive than electrons (by nearly a factor of 2000). On the other hand, nuclear diameters are very small in comparison to atomic diameters  $(10^{-13}$  cm versus  $10^{-8}$  cm). Thus it can be deduced that the density of nuclear matter is very high  $(\sim 10^{14} \text{ g/cm}^3)$  and that the rest of the atom (electron cloud) is mostly empty space.

### 2. Terminology and Notation

An atomic nucleus is characterized by the number of neutrons and protons it contains. The number of protons determines the *atomic number* of the atom, Z. As mentioned earlier, this determines also the number of orbital electrons in the electrically neutral atom and therefore the *chemical element* to which the atom belongs.

The total number of nucleons is the *mass number* of the nucleus, A. The difference, A-Z, is the *neutron number*, N. The mass number A is approximately equal to, but not the same as the *atomic weight* (AW) used in chemistry. The latter is the average weight of an atom of an element in its natural abundance (Appendix A).

Table 1-4
Basic Properties of Nucleons and Electrons

Particle	Charge†	Mass (u)	Mass (MeV)	
Proton	+1	1.007593	938.211	
Neutron	0	1.008982	939.505	
Electron	<b>-1</b>	0.000548	0.511	

<sup>†</sup>One unit of charge is equivalent to  $1.602 \times 10^{-19}$  coulombs.

The notation now used to summarize atomic and nuclear composition is  ${}_{Z}^{A}X_{N}$ , where X represents the chemical element to which the atom belongs. For example, an atom comprised of 53 protons, 78 neutrons (and thus 131 nucleons), and 53 orbital electrons represents the element iodine and is symbolized by  ${}_{33}^{13}I_{78}$ . Since all iodine atoms have atomic number 53, either the "I" or the "53" is redundant, and the "53" can be omitted. The neutron number, 78, can be inferred from the difference 131-53 and also can be omitted. Therefore a shortened but still complete notation for this atom is  ${}^{131}I$ . An acceptable alternative in terms of medical terminology is I-131. Obsolete forms (often found in older texts) include  ${}^{131}$ ,  ${}_{131}I$ , and  ${}^{1}I_{131}I$ .

#### 3. Nuclear Families

Nuclear species are sometimes grouped into families having certain common characteristics. A *nuclide* is characterized by an exact nuclear composition, including the mass number A, atomic number Z, and arrangement of nucleons within the nucleus. To be classified as a nuclide, the species must have a "measurably long" existence, which for current technology means a lifetime greater than about  $10^{-12}$  sec. For example,  $^{12}$ C,  $^{16}$ O, and  $^{131}$ I are nuclides.

Nuclides that have the same atomic number Z are called *isotopes*. Thus  $^{125}\text{I}$ ,  $^{127}\text{I}$ , and  $^{131}\text{I}$  are isotopes of the element iodine. Nuclides with the same mass number A are *isobars*—e.g.,  $^{131}\text{I}$ ,  $^{131}\text{Xe}$ ,  $^{131}\text{Cs}$ . Nuclides with the same neutron number N are *isotones*—e.g.,  $^{131}_{53}\text{I}_{78}$ ,  $^{132}_{54}\text{Xe}_{78}$ ,  $^{133}_{55}\text{Cs}_{78}$ . A mnemonic device for remembering these relationships is that isotopes have the same number of protons, isotones the same number of neutrons, and isobars the same mass number.

#### 4. Nuclear Forces and Energy Levels

Nucleons within the nucleus are subject to two kinds of forces. Repulsive coulombic or electrical forces exist between positively charged protons. These are counteracted by very strong forces of attraction, called exchange forces, between any two nucleons. Exchange forces are effective only over very short distances, and their effects are seen only when nucleons are very close together, as they are in the nucleus. Exchange forces hold the nucleus together against the repulsive coulombic forces between protons.

Nucleons move about within the nucleus in a very complicated way under the influence of these forces. One model of the nucleus, called the *shell model*, portrays the nucleons as moving in "orbits" about one another in a manner similar to that of orbital electrons moving about the nucleus in the Bohr atom. Only a limited number of motions are allowed, and these are determined by a set of nuclear quantum numbers.

The most stable arrangement of nucleons is called the *ground* state. Other arrangements of the nucleons fall into two categories:

- 1. Excited states are arrangements that are so unstable that they have only a transient existence before transforming into some other state.
- 2. Metastable states are also unstable, but they have relatively long lifetimes before transforming into another state. These are also called *isomeric* states.

The dividing line for lifetimes between excited and metastable states is about  $10^{-12}$  sec. This is not a long time according to everyday standards, but it is "relatively long" by nuclear standards. Some metastable states are quite long-lived, i.e., average lifetimes of several hours. (The prefix *meta* derives from the Greek word for "almost.") Because of this, metastable states are considered to have separate identities and are classified themselves as nuclides. Two nuclides that differ from one another in that one is a metastable state of the other are called *isomers*.

In nuclear notation, excited states are identified by an asterisk ( $^{A}X^{*}$ ) and metastable states by the letter m ( $^{Am}X$  or X-Am).† Thus  $^{99m}$ Tc (or Tc-99m) represents a metastable state of  $^{99}$ Tc, and  $^{99m}$ Tc are isomers.

Nuclear transitions between different nucleon arrangements involve discrete and exact amounts of energy, as do the rearrangements of orbital electrons in the Bohr atom. A nuclear energy level diagram is used to identify the various excited and metastable states of a nuclide and the energy relationships among them. Figure 1–6 shows a partial diagram for <sup>131</sup>Xe.‡ The bottom line represents the ground state, and other lines represent excited or metastable states. Metastable states usually are indicated by somewhat heavier lines. The vertical distances between lines are proportional to the energy differences between levels. A transition from a lower to a higher state requires an energy input of some sort, e.g., a photon or particle striking the nucleus. Transitions from higher to lower states result in the release of energy, which is given to emitted particles or photons.

#### 5. Nuclear Emissions

The energy released in a nuclear transformation to a more stable state may appear as a photon of electromagnetic radiation. Photons of nuclear origin are called  $\gamma$  rays (gamma rays). The energy difference between the states involved in the transition determines the  $\gamma$ -ray energy. For example, in Figure 1–6 a transition from the level marked 0.364 MeV to the ground state would produce a 0.364 MeV  $\gamma$  ray. A transition from the 0.364 MeV level to the 0.080 MeV level would produce a 0.284 MeV  $\gamma$  ray.

As an alternative to emitting a  $\gamma$  ray, the nucleus may transfer the energy to an orbital electron and emit the electron instead of a photon. This process, which is similar to the Auger effect in x-ray emission (Section C.3), is called *internal conversion*. It is discussed in detail in Chapter 2, Section E.

<sup>†</sup>The notation  ${}^{A}X^{m}$  is common in Europe (e.g.,  ${}^{99}\text{Tc}^{m}$ ).

<sup>‡</sup>Actually, these are the excited and metastable states formed during radioactive decay by  $\beta$ -emission of <sup>131</sup>I (Chapter 2, Section D).

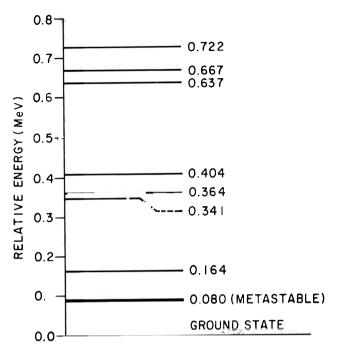


Fig. 1-6. Partial nuclear energy level diagram for <sup>131</sup>Xe nucleus. Vertical axis represents energy differences between nuclear states (or "arrangements"). Going up the scale requires energy input. Coming down the scale results in the emission of nuclear energy. Heavier lines indicate metastable states.

#### 6. Nuclear Binding Energy

When the mass of an atom is compared to the sum of the masses of its individual components (protons, neutrons, and electrons), it is always found to be less by some amount,  $\Delta m$ . This mass deficiency, expressed in energy units, is called the *binding energy*  $E_B$  of the atom:

$$E_B = \Delta mc^2 \tag{1--4}$$

For example, consider an atom of <sup>12</sup>C. This atom is comprised of six protons, six electrons, and six neutrons, and its mass is precisely 12.0 u (by definition of the universal mass unit u). The sum of the masses of its components is

electrons	$6 \times 0.000055$	u =	0.000330 u
protons	$6 \times 1.007593$	u =	6.045558 u
neutrons	$6 \times 1.008982$		
			12.099780 u

Thus  $\Delta m = 0.09978$  u. Since 1 u = 931.5 MeV, the binding energy of a  $^{12}$ C atom is  $0.099780 \times 931.5 = 92.95$  MeV.

The binding energy is the minimum amount of energy required to overcome the forces holding the atom together in order to separate it completely into its individual components. Some of this represents the binding energy of orbital electrons, i.e., the energy required to strip the orbital electrons away from the nucleus; however, comparison of the total binding energy of a <sup>12</sup>C atom with the K shell binding energy of carbon (Appendix A) indicates that most of this energy is *nuclear binding energy*, i.e., energy required to separate the nucleons.

Nuclear processes that result in the release of energy (e.g.,  $\gamma$ -ray emission) always *increase* the binding energy of the nucleus. Thus a nucleus emitting a 1 MeV  $\gamma$  ray would be found to weigh *less* (by the mass equivalent of 1 MeV) after the  $\gamma$  ray was emitted than before. In essence, mass is converted to energy in the process.

#### 7. Characteristics of Stable Nuclei

Not all combinations of protons and neutrons produce stable nuclei. Some are unstable, even in their ground states. An unstable nucleus emits particles and/or photons to transform itself into a more stable nucleus. This is the process of radioactive disintegration or radioactive decay, discussed in Chapter 2. A survey of the general characteristics of naturally occurring stable nuclides provides clues to the factors that contribute to nuclear instability and thus to radioactive decay.

A first observation is that there are favored neutron-to-proton ratios among stable nuclides. Figure 1–7 is a plot of the stable nuclides according to their neutron and proton numbers. For example, the nuclide  ${}^{12}_{6}C$  is represented by a dot at the point Z=6, N=6. The stable nuclides are clustered around an imaginary line called the *line of stability*. For light elements, the line corresponds to  $N\approx Z$ , i.e., about equal numbers of protons and neutrons. For heavy elements, it corresponds to  $N\approx 1.5Z$ , i.e. about 50 percent more neutrons than protons. The line of stability ends at  $^{209}$ Bi. All heavier nuclides are unstable.

In general, there is a tendency toward instability in systems comprised of large numbers of identical particles confined in a small volume. This explains the instability of very heavy nuclei. It also explains why, for light elements, stability is favored by more or less equal numbers of neutrons and protons rather than grossly unequal numbers. A moderate excess of neutrons is favored among heavier elements because neutrons provide only exchange forces (attraction), whereas protons provide both exchange forces and coulombic forces (repulsion). Exchange forces are effective over very short distances and thus affect only "close neighbors" in the nucleus, whereas the repulsive coulombic forces are effective over much greater distances. Thus an excess of neutrons is required in heavy nuclei to overcome the long-range repulsive Coulombic forces between a large number of protons.

Nuclides that are not close to the line of stability are likely to be unstable.

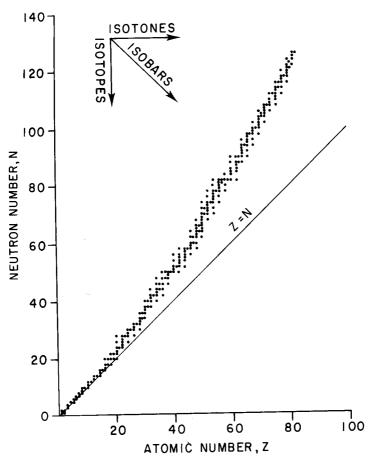


Fig. 1-7. Atomic number Z versus neutron number N for the stable nuclides. They are clustered around an imaginary line called the line of stability,  $N \approx Z$  for light elements;  $N \approx 1.5Z$  for heavy elements.

Unstable nuclides lying above the line of stability are said to be "proton deficient," while those lying below the line are "neutron deficient." Unstable nuclides generally undergo radioactive decay processes that transform them into nuclides lying closer to the line of stability, as discussed in Chapter 2.

From Figure 1-7 it can be seen that there are often many stable isotopes of an element. Isotopes fall on vertical lines in the diagram. For example, there are ten stable isotopes of tin  $(Sn, \dagger Z = 50)$ . There may also be several stable

<sup>†</sup>While most element symbols are simply one- or two-letter abbreviations of their (English) names, 11 symbols derive from Latin or Greek names of metals known for over two millennia: antimony (stibium, Sb); copper (cuprum, Cu); gold (aurum, Au); iron (ferrum, Fe); lead (plumbum, Pb); mercury (hydrargyrum, Hg); potassium (kalium, K); silver (argentum, Ag); sodium (natrium, Na); tin (stannum, Sn); tungsten (wolfram, W).

isotones. These fall along horizontal lines. In relatively few cases, however, is there more than one stable isobar (isobars fall along descending 45° lines on the graph), reflecting the existence of several modes of "isobaric" radioactive decay that permit nuclides to transform along isobaric lines until the most stable isobar is reached. This will be discussed in detail in Chapter 2.

One also notes among the stable nuclides a tendency to favor even numbers. For example, there are 165 stable nuclides with both even numbers of protons and even numbers of neutrons. Examples are <sup>4</sup><sub>2</sub>He and <sup>12</sup><sub>6</sub>C. There are 109 "even-odd" nuclides, with even numbers of protons and odd numbers of neutrons or vice versa. Examples are <sup>9</sup><sub>4</sub>Be and <sup>11</sup><sub>5</sub>B. However, there are only four stable "odd-odd" nuclides: <sup>2</sup><sub>1</sub>H, <sup>6</sup><sub>3</sub>Li, <sup>15</sup><sub>5</sub>B, and <sup>14</sup><sub>7</sub>N. The stability of even numbers reflects the tendency of nuclei to achieve stable arrangements by the "pairing up" of nucleons in the nucleus.

Another measure of relative nuclear stability is nuclear binding energy, since this represents the amount of energy required to break the nucleus up into its separate components. Obviously, the greater the number of nucleons, the greater the total binding energy. Therefore a more meaningful parameter is the binding energy per nucleon,  $E_B/A$ . Higher values of  $E_B/A$  are indicators of greater nuclear stability.

Figure 1-8 is a graph of  $E_B/A$  versus A for the stable nuclides. Binding

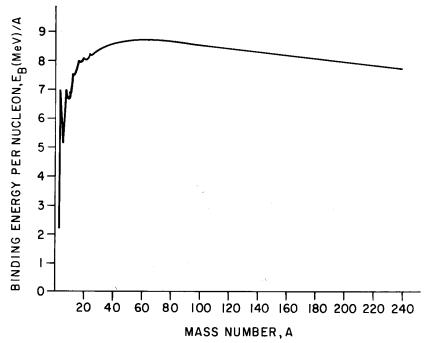


Fig. 1–8. Binding energy per nucleon versus mass number for the stable nuclides. [Adapted with permission from Evans RD, The Atomic Nucleus. New York, McGraw-Hill, 1972, p. 299].

energy is greatest ( $\sim$ 8 MeV/nucleon) for nuclides of mass number  $A \approx 60$ . It decreases slowly with increasing A, indicating the tendency toward instability for very heavy nuclides. Finally, there are a few peaks in the curve representing very stable light nuclides, including  $^4_2$ He,  $^{12}_6$ C, and  $^{16}_8$ O. Note that these are all "even—even" nuclides.

#### REFERENCES

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