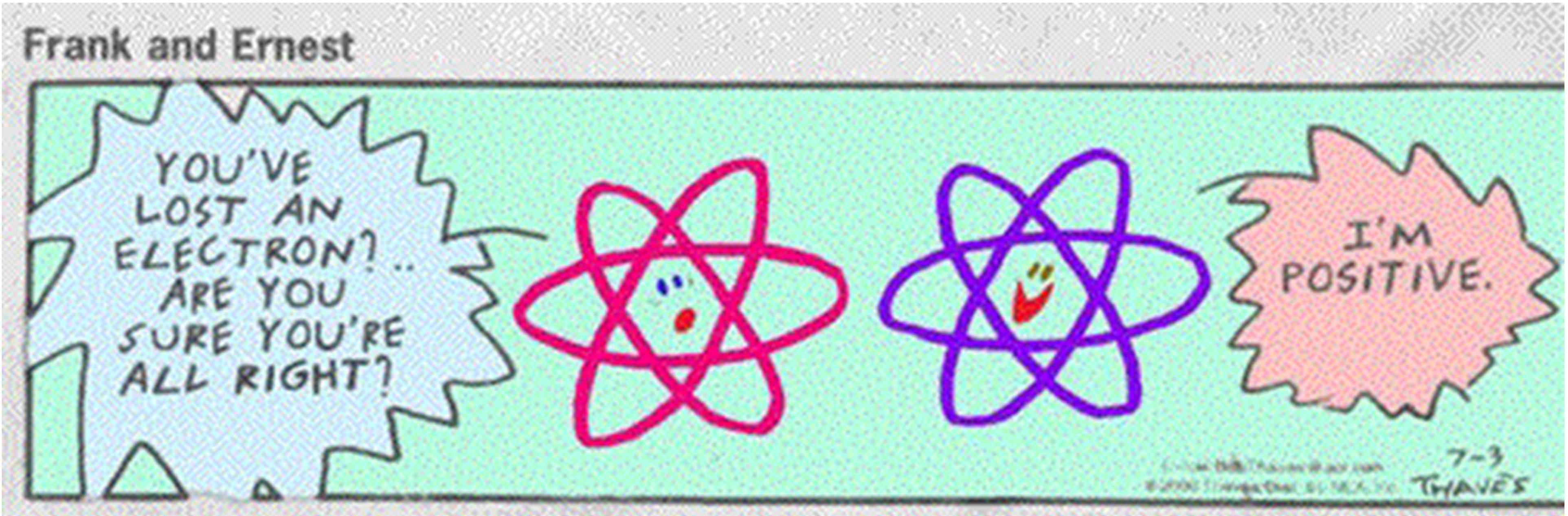
University of Tripoli
Department of Nuclear Engineering
NE 639

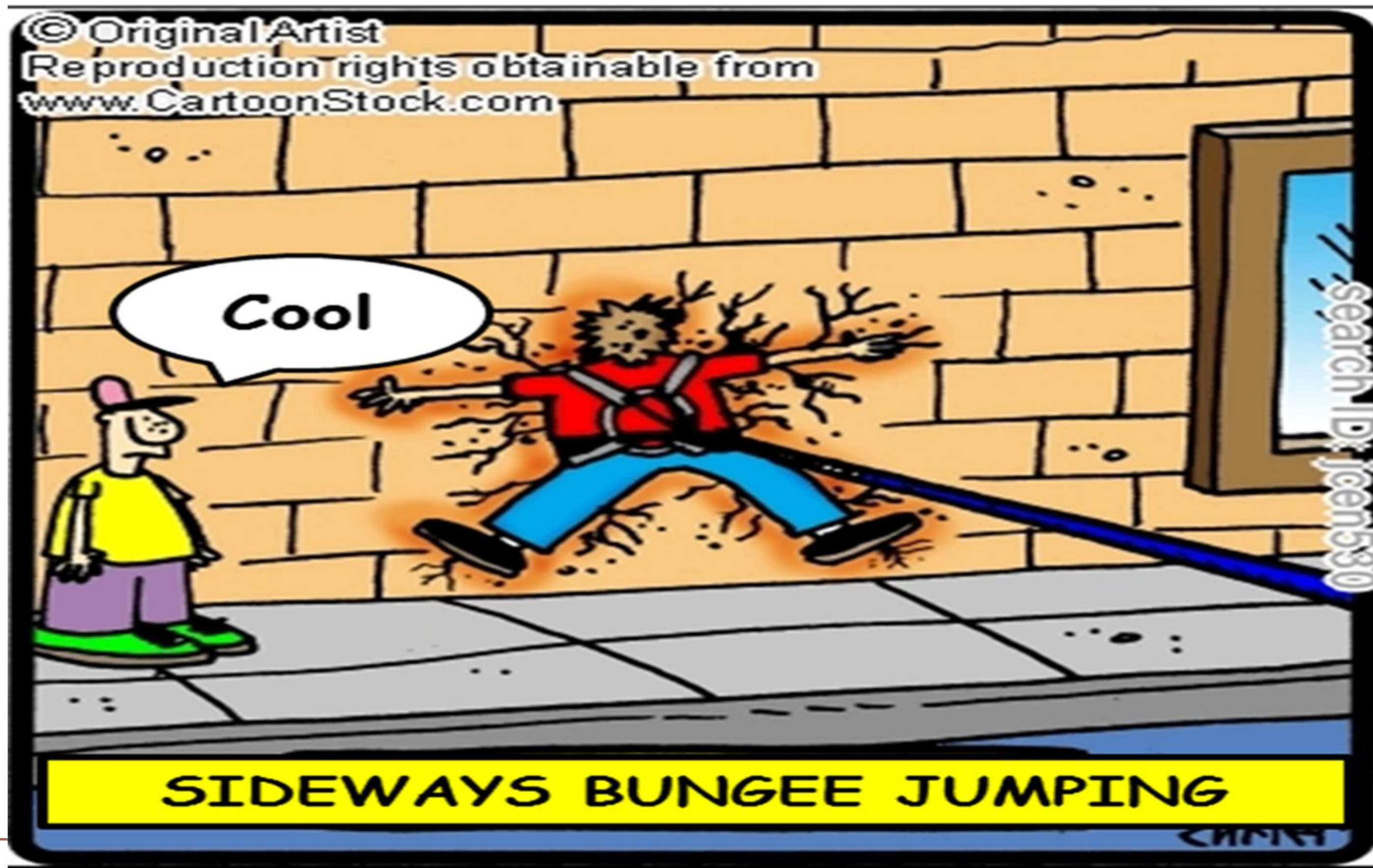
THE PHYSICS OF RADIONUCLIDE IMAGING



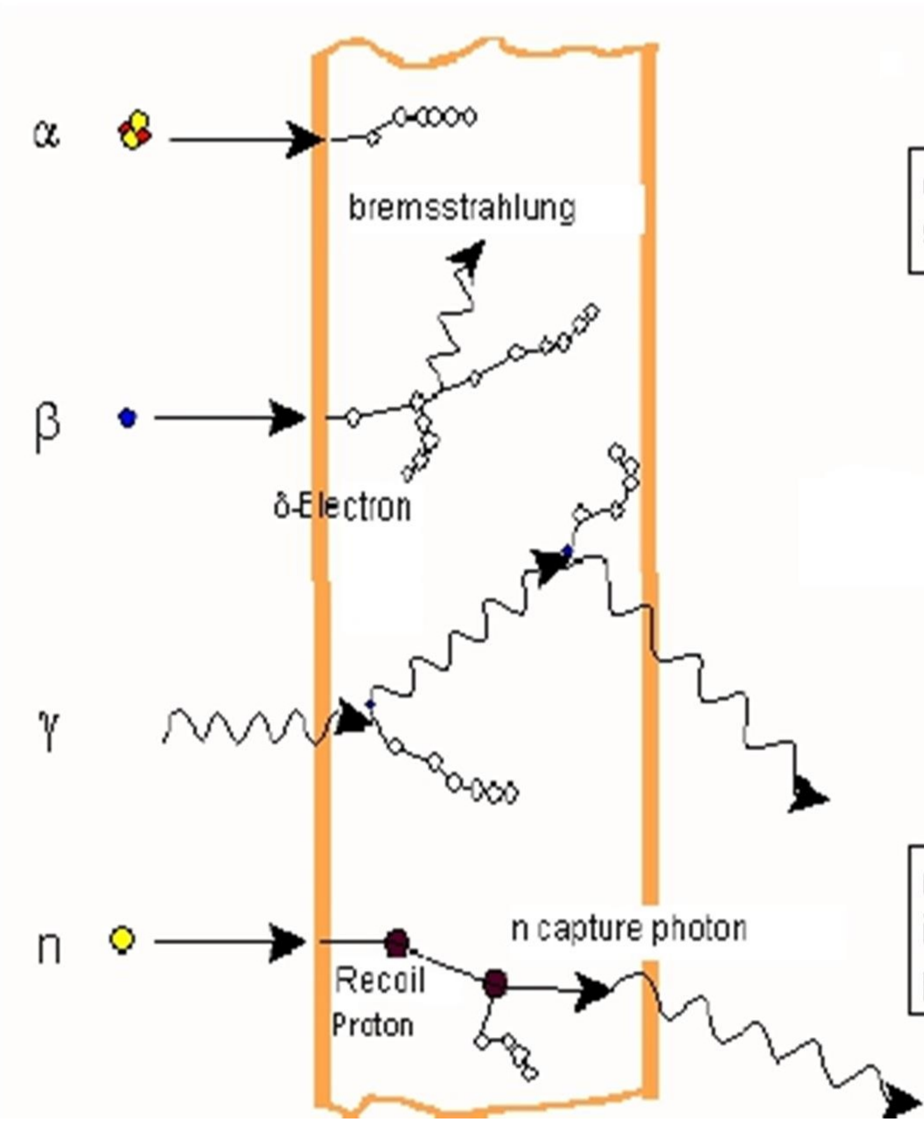
Interaction of Radiation with matter

Introduction

- In order to understand the basic concepts in X-ray transmission and gamma-ray emission imaging, we need to understand the underlying interaction processes of the photons with the object to be imaged and with the photon sensing detector.

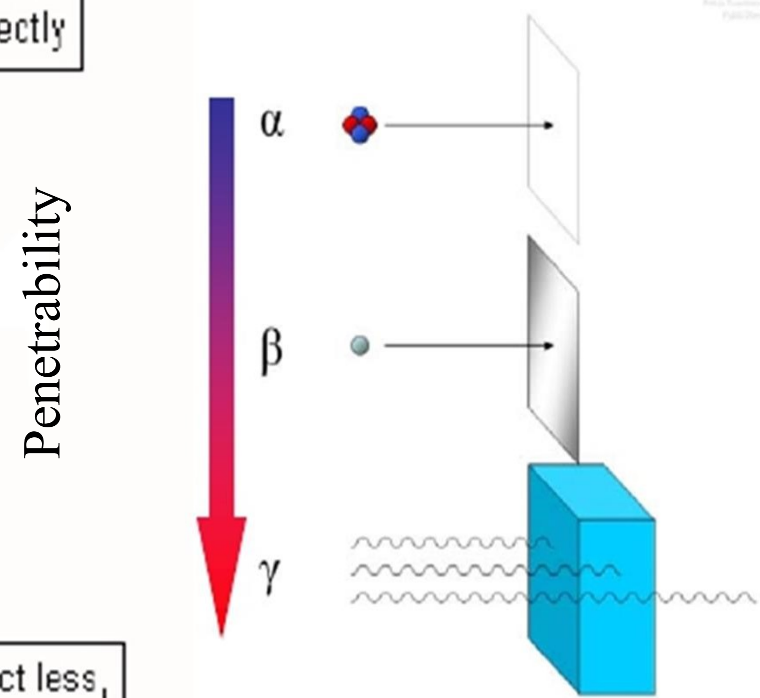


Interactions of Radiation with Matter

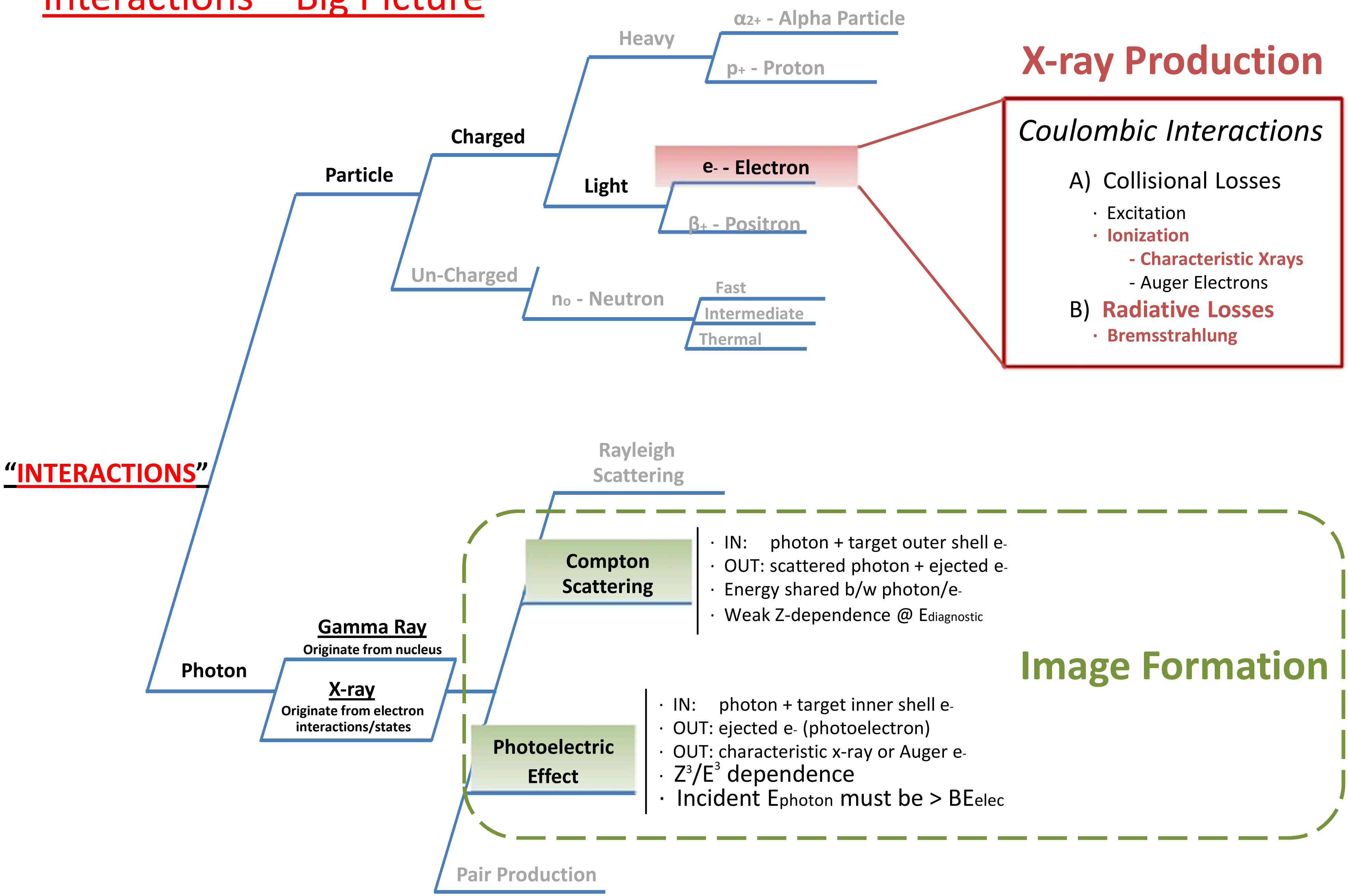


charged particles interact strongly and ionize directly

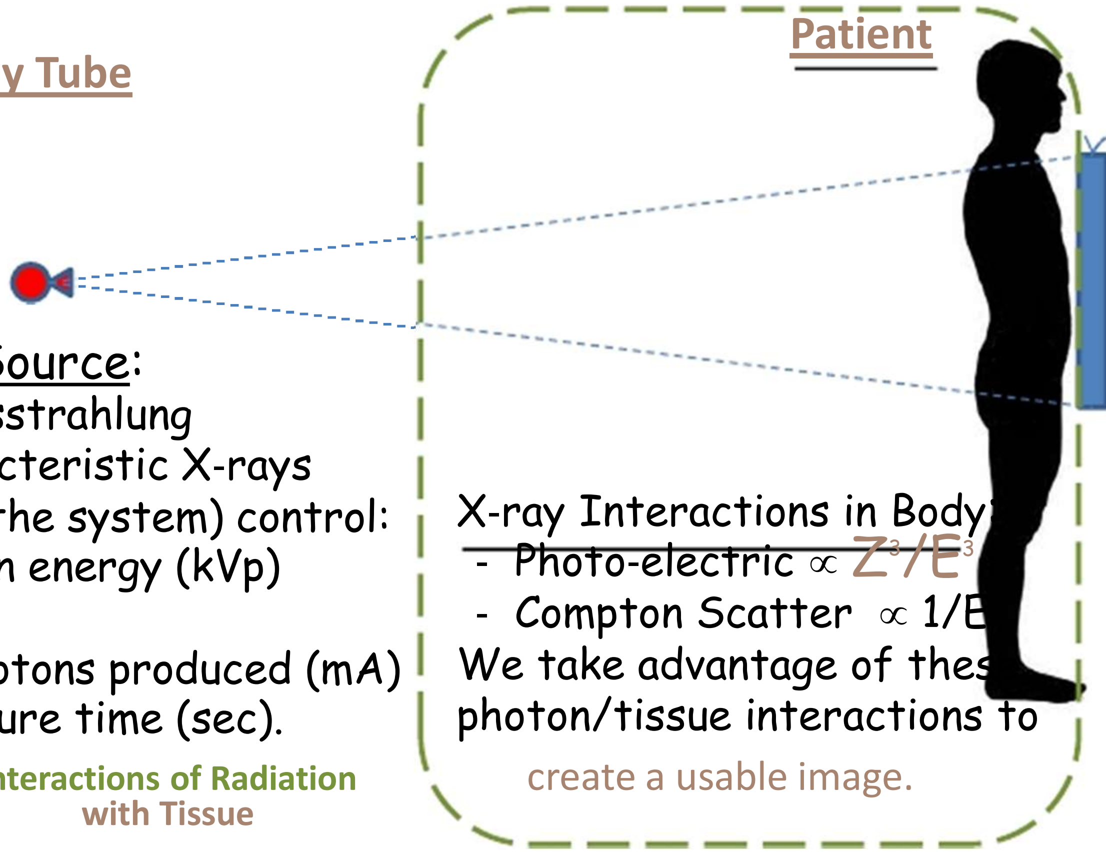
neutral particles interact less, ionize indirectly and penetrate farther



Interactions – Big Picture



Global Perspective



X-ray Source:
- Bremsstrahlung
- Characteristic X-rays

We (or the system) control:
- photon energy (kVp)
-

# of photons produced (mA)
- exposure time (sec).

Interactions of Radiation with Tissue

X-ray Interactions in Body:
- ~~Photo-electric ∝ Z^3/E^3~~
- Compton Scatter ∝ 1/E

We take advantage of thes photon/tissue interactions to create a usable image.

Image Receptor:
- A) Analog Detector
  * Screen-film
- B) Digital Detector
  * Computed Radiography
  * In-direct Digital
  * Direct Digital

- This is classified as *transmission* imaging (i.e. photons transmit through patient)
- This is a 2D PROJECTION of 3D anatomy!
  - Anatomical overlay (we lack some spatial information)
  - SOLUTION: Different views (or a different modality)

E – photon energy
Z – atomic number

X-ray Absorption and Scatter

Four major interactions are of importance to diagnostic radiology and nuclear medicine, each characterized by a probability (or "cross-section") of interaction

- Classical (Rayleigh or elastic) scattering
- Compton scattering
- Photoelectric effect
- Pair production (not in diagnostic energy range, incident x-ray must have energy > 1.02 MeV)

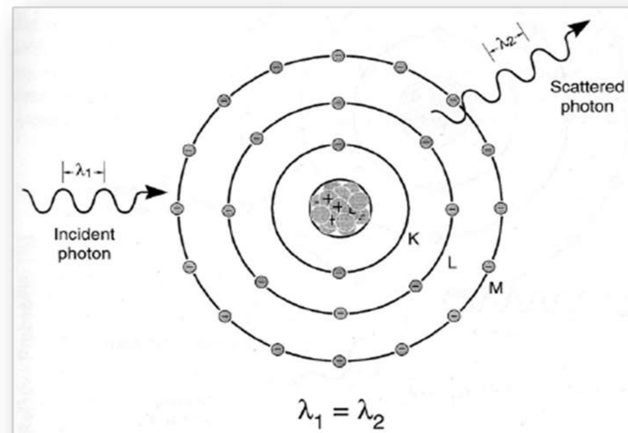
E_γ = energy of incident photon

E_{BE} = binding energy of orbital electron

Classical (Rayleigh or elastic) Scattering($\sigma_R \propto \rho \frac{Z^2}{E_\gamma}$)

$E_\gamma << E_{BE}$

- Incident photon interacts with and excites the total atom (electrons in the atom oscillate in phase)
- Occurs at very low energies (15 to 30 keV),
  increases in probability with decreasing energy.
- No ionization takes place and no loss of energy
- The photon is scattered (re-emitted)
  in a range of different directions
  , but close to that of the incident photon



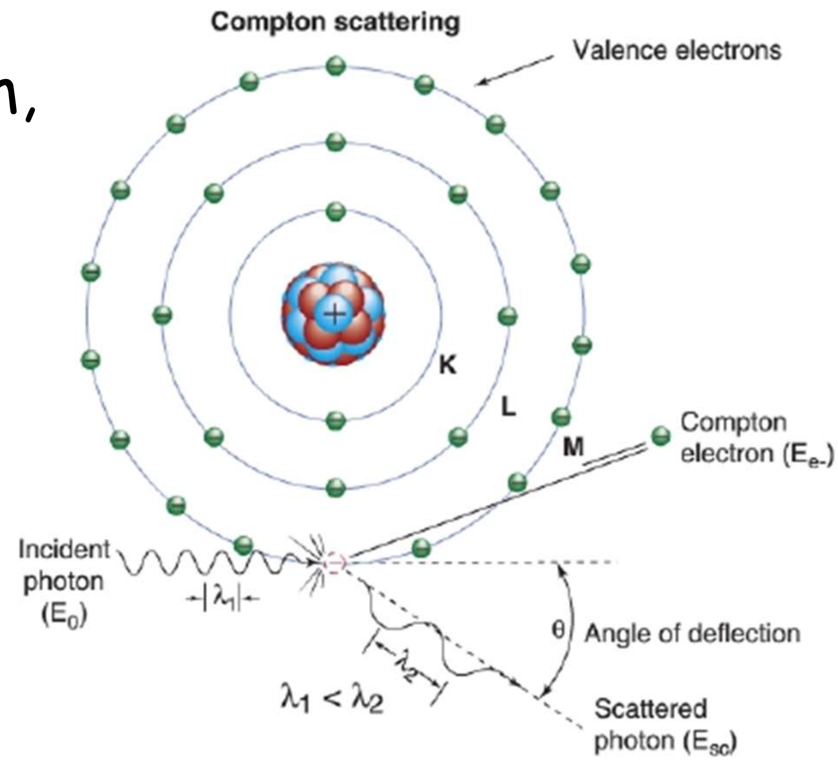
- *Detection of scattered photo has negative effect on image quality*
- Relatively infrequent probability (~ 5%)
  for energies > 70 keV (diagnostic energy range)
- Accounts for ~10% of measured photons at 30 keV
  (mammography energy range, Only important for low energies (<20 KeV,)

Compton Scattering

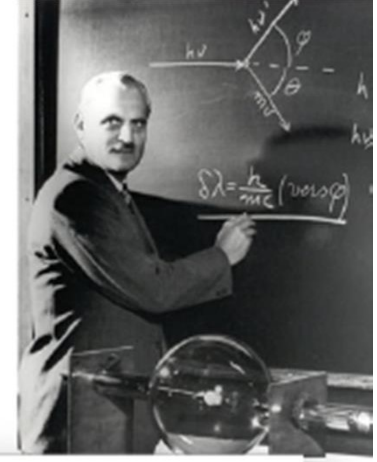
E_γ >> E_{BE}

- Dominant interaction of x-rays with soft tissue in the diagnostic range and beyond (approx 30 keV - 30MeV)

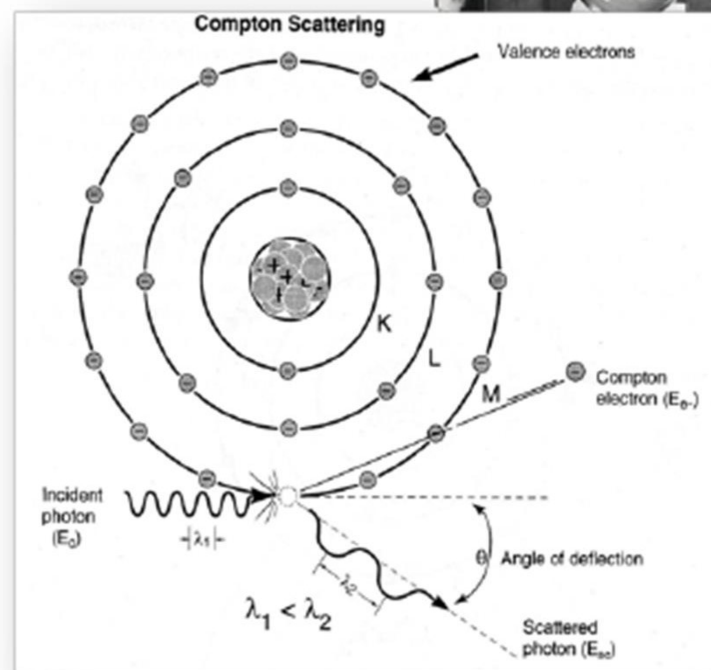
- Interaction of incident photon and an outer shell electron
- Results in ionization of the atom, a scattered photon, and the ejected electron
- Ejected (Compton) electron loses its energy via excitation and ionization



Compton Scattering



- Scattering of a photon by a (free) electron that leads to a moving electron and a lower energy photon
- Secondary photon degrades transmission image

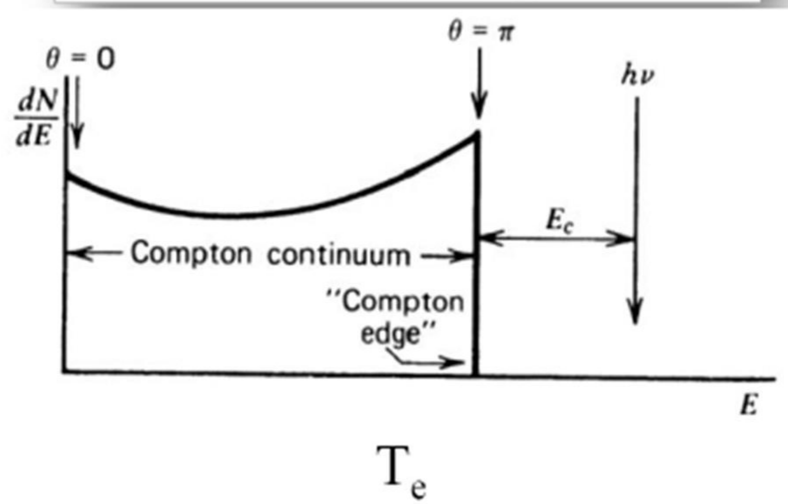


$$E_{\gamma}' = \frac{E_{\gamma}}{1 + E_{\gamma}/E_0(1-\cos\theta)}$$

$$E_e = E_{\gamma}\frac{1-\cos\theta}{E_0/E_{\gamma}+1-\cos\theta}$$

$$\sigma_{CS} \propto \rho_e/E \qquad \rho_e = \rho \times N_A \times Z/A$$

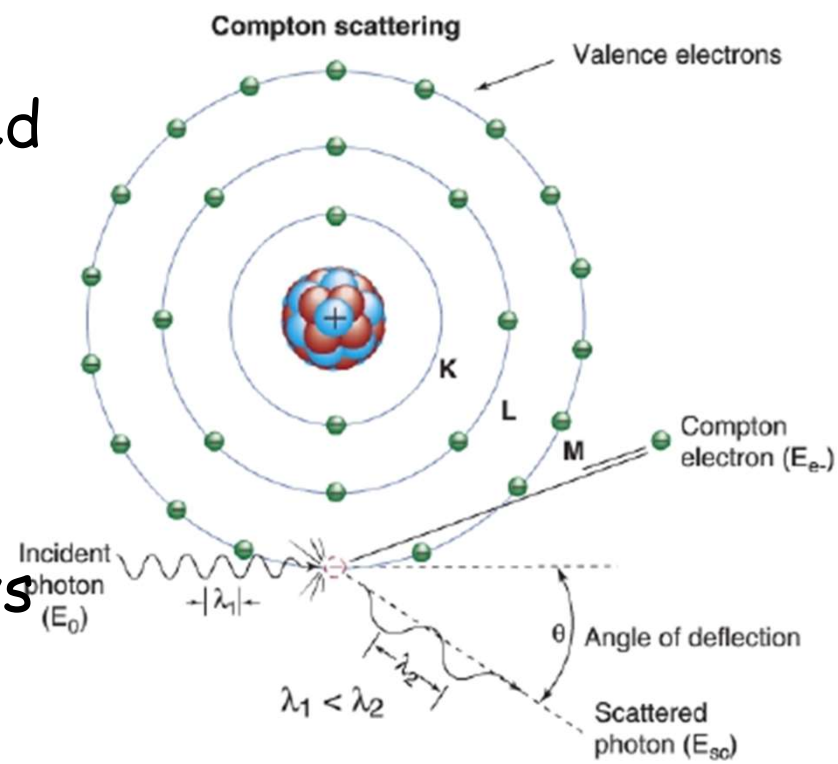
ρ_e: Electron density



Compton Scattering

$E_\gamma >> E_{BE}$ (*outer shell electron interaction*)

- Original photon 'deflected' which in turn decreases subject contrast
- As incident E_0 increases, the photon & electron are scattered in more forward direction, increasing likelihood of being measured by the imaging detector (decreased contrast)
- Compton scatter is greatest contributor of scattered x-rays in diagnostic imaging



Compton Scattering

$E_\gamma >> E_{BE}$ *(outer shell electron interaction)*

- Conservation of energy & momentum:

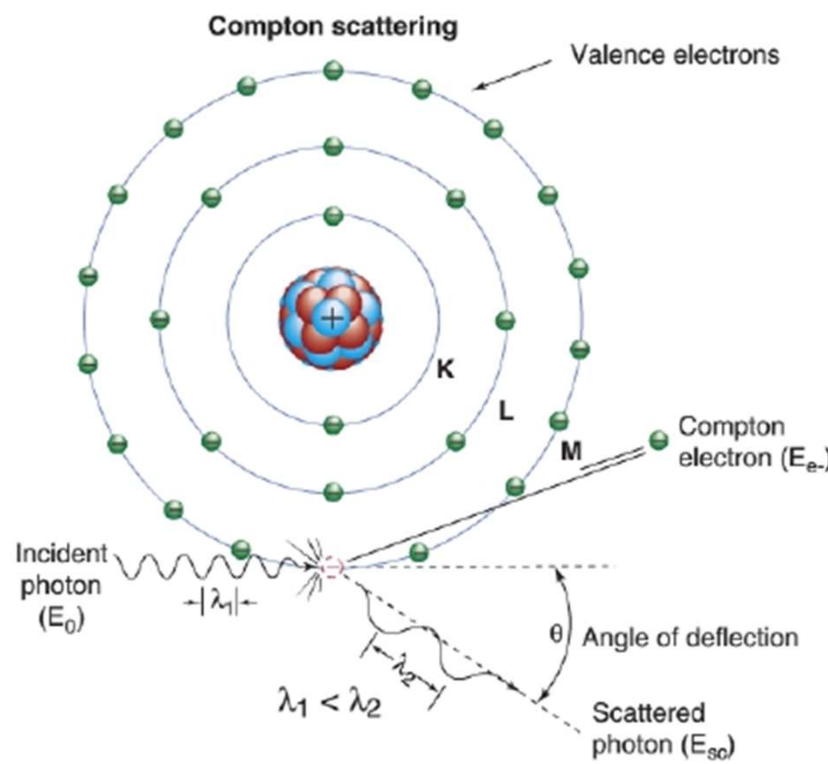
  $E_o = E_{sc} + E_e$

  E_o = incident photon energy

  E_{sc} = scattered photon energy

  E_e = ejected electron energy

- At diagnostic energies (20-100 keV) most energy transferred to the scattered photon

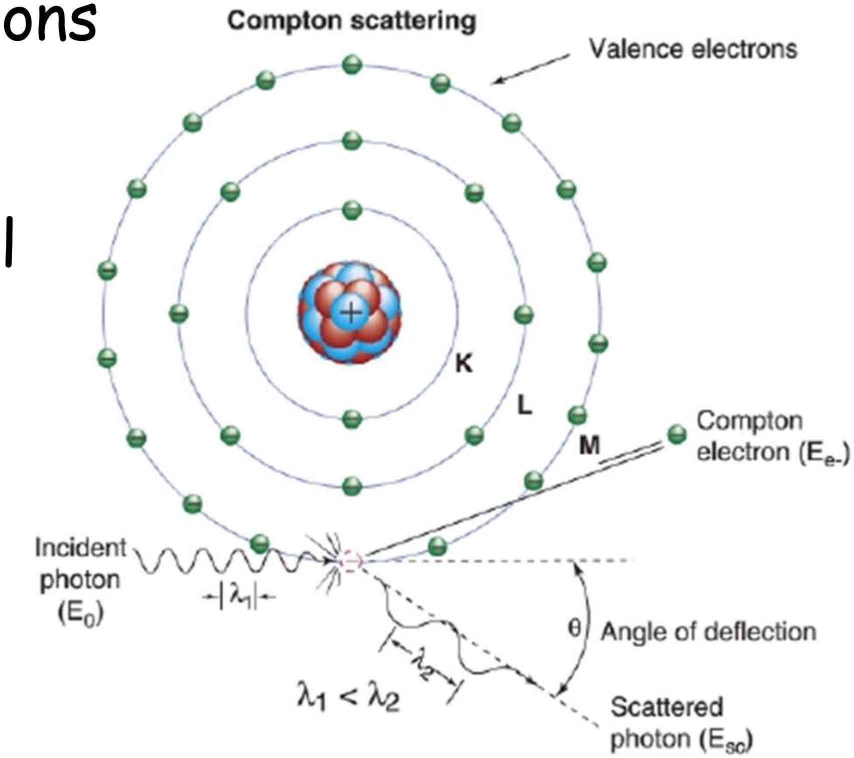


Compton Scattering

$E_\gamma >> E_{BE}$ (*outer shell electron interaction*)

- Probability of Compton interaction is primarily dependent on electron density, increasing with increased electron density

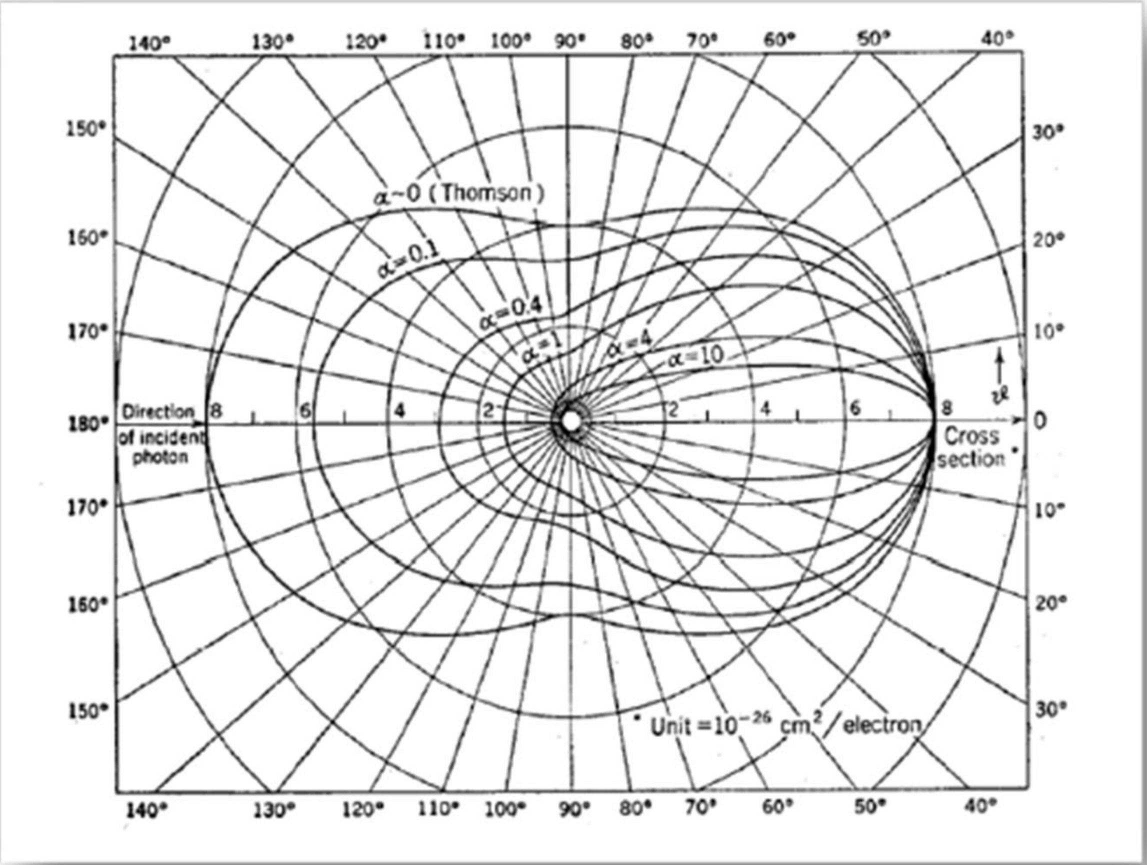
- In tissue, the number of electrons per gram is fairly constant, therefore the probability is roughly proportional to material density (ρ)

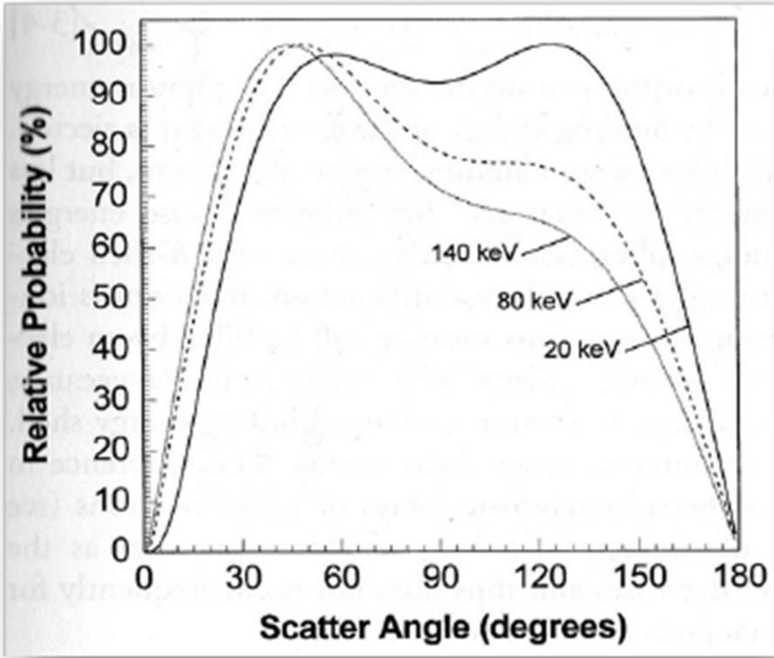


Klein-Nishina Formula

Differential cross section per electron

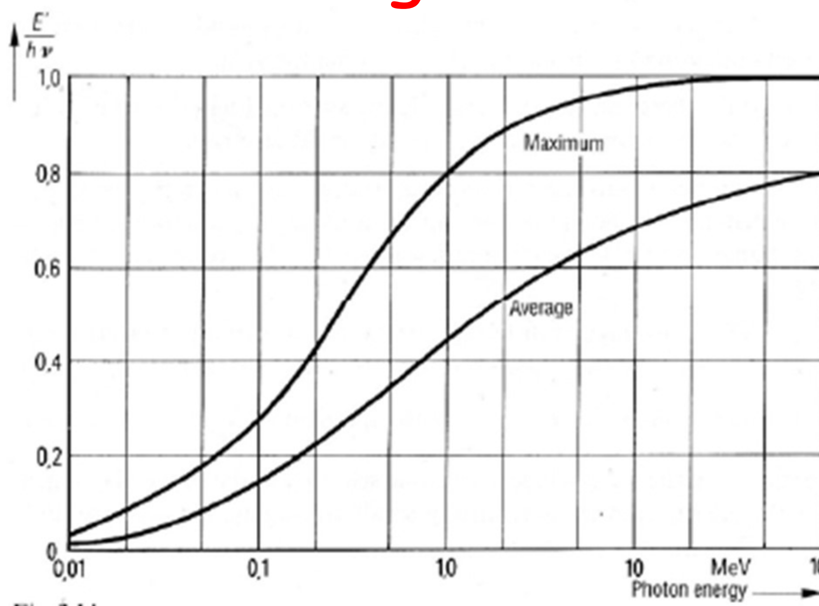
$$\frac{d\sigma}{d\Omega} = Zr_0^2\left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2\left(\frac{1+\cos^2\theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)(1+\alpha(1-\cos\theta))}\right) \qquad \alpha = \frac{h\nu}{m_e c^2}$$

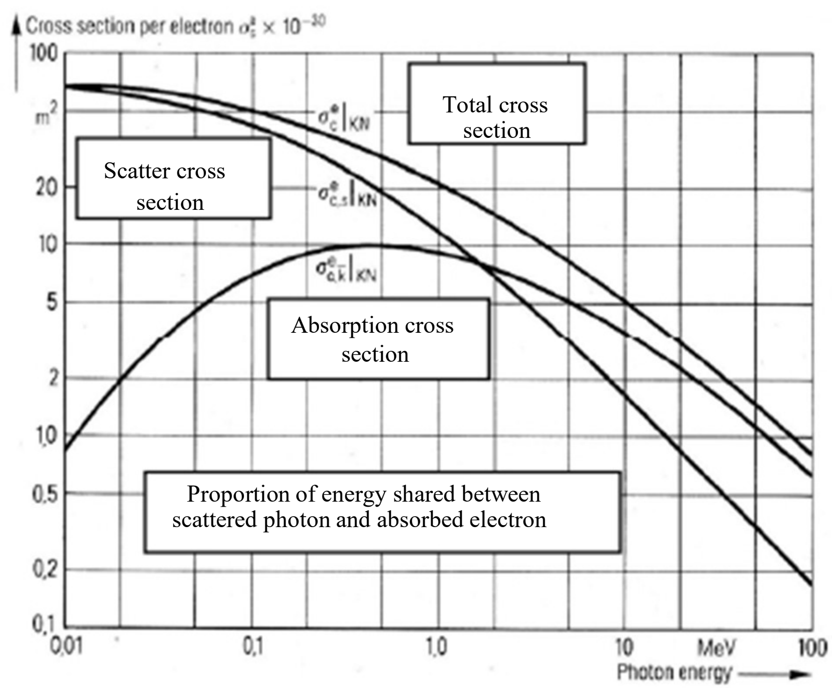




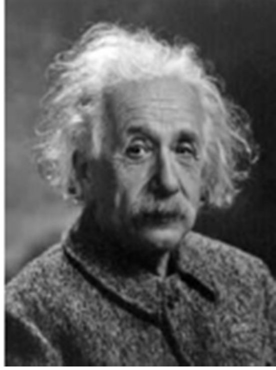
Energy-Sharing in Compton Scattering

- We need to distinguish between the maximum and average energy that is transferred to the Compton (recoil) electron
- Higher-energy photons lose on average more energy to the recoil electron during Compton scattering
- For energies below ~1.5MeV, more energy remains in the photon than transferred to the electron.





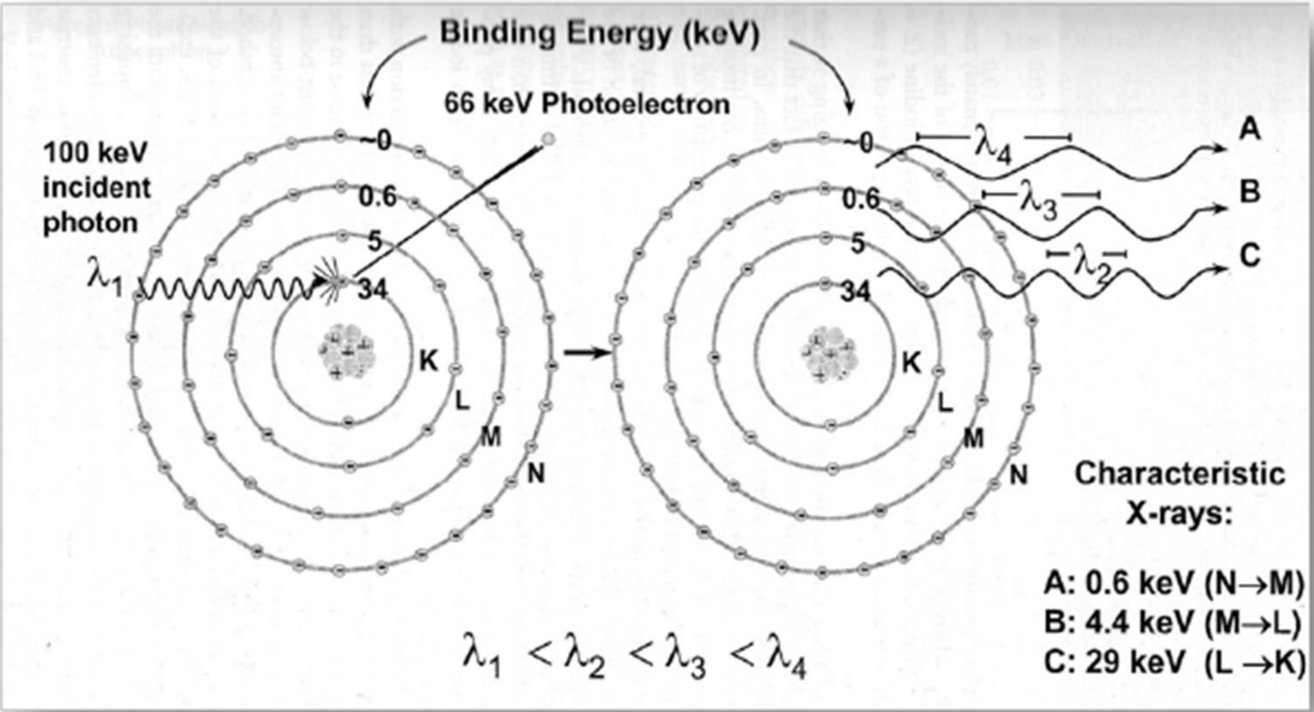
Photoelectric Absorption

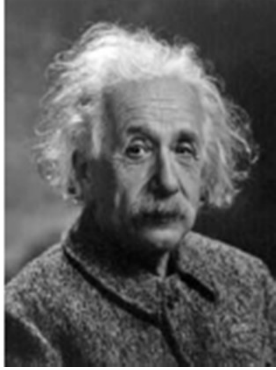


- Entire photon energy is transferred to a bound (most likely K-) electron
- Only secondary particle is short-ranged electron, therefore no transmission image degradation

$$E_{e^-} = h\nu - E_b, \quad E_\gamma = h\nu$$

$$\sigma_{PE} \propto \rho \frac{Z^4}{E_\gamma^{3.5}}$$





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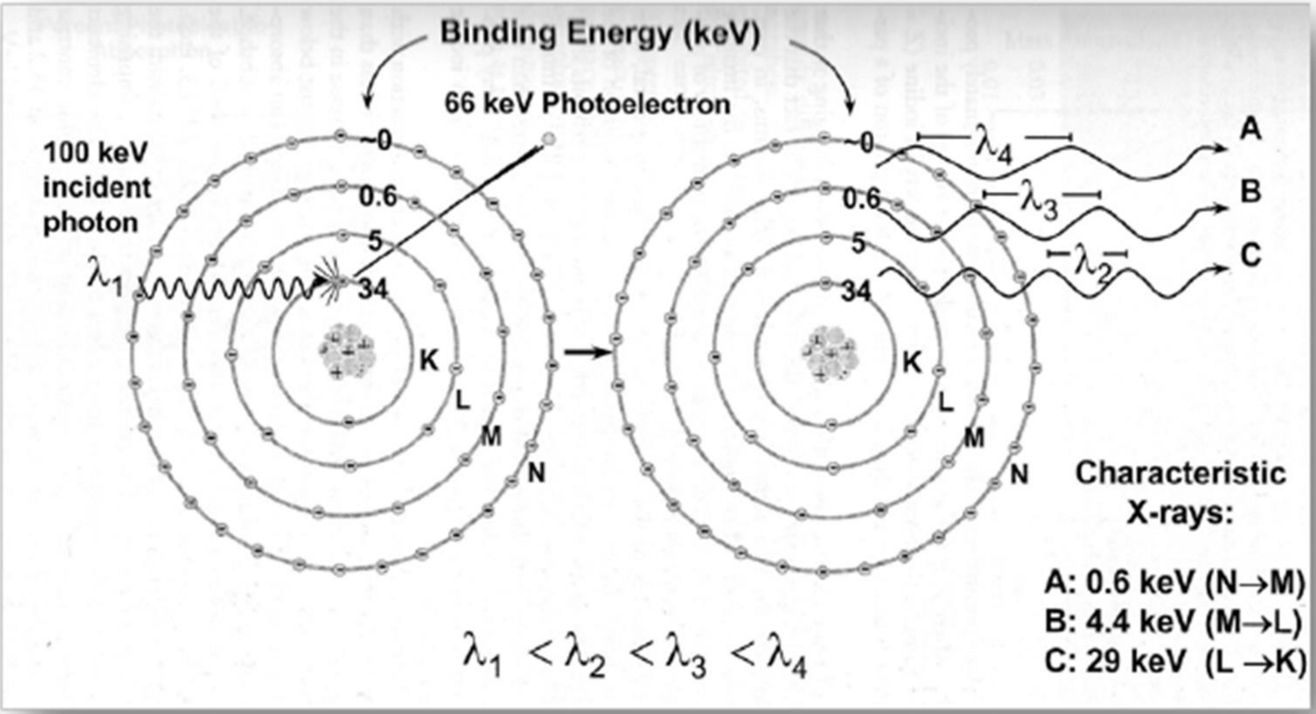


Photoelectric Effect (PE)

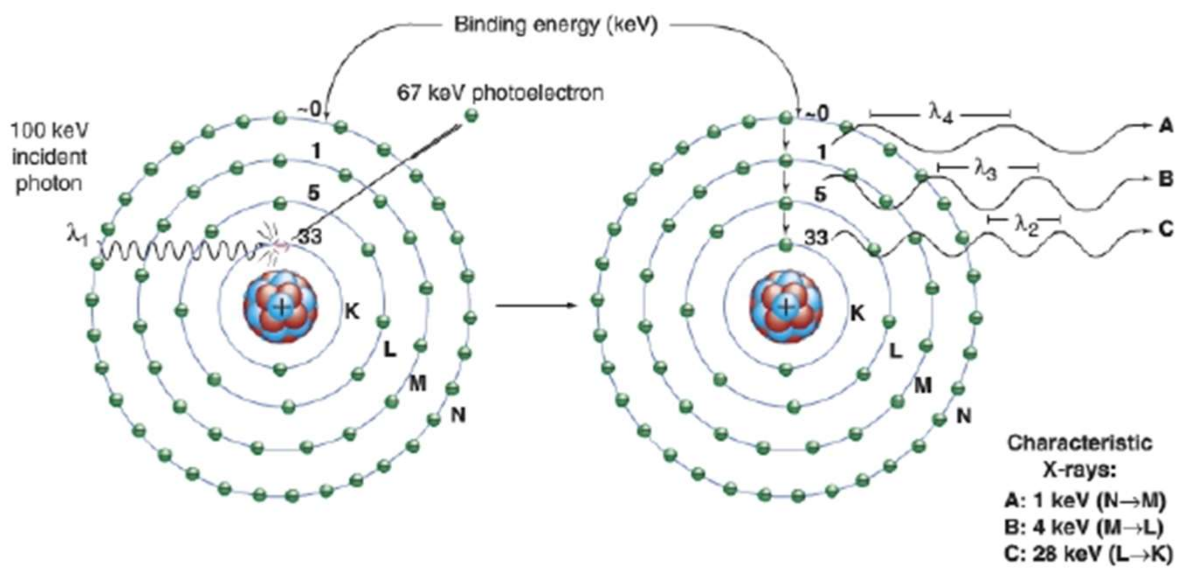
$E_\gamma >> E_{BE}$

- <u>Interaction of incident photon with inner shell electron</u>

- Results in a photoelectron and characteristic x-ray (or Auger electron)

- All incident photon energy (E_o) is transferred to the ejected photoelectron (E_e)

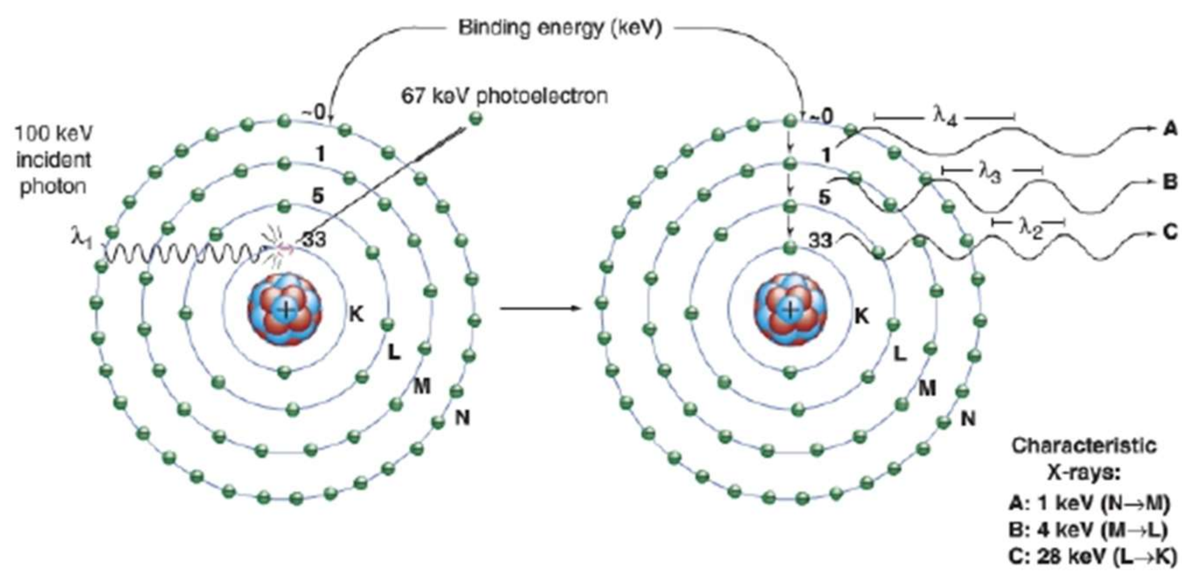
- $E_e = E_o - E_{BE}$



Photoelectric Effect (PE)

$E_{\gamma} >> E_{BE}$ *(inner shell electron interaction)*

- The empty electron shell immediately fills with an electron from an outer orbital resulting in the emission of characteristic x-rays (or Auger electron)
- The energy of the characteristic x-ray emitted (E□) is the difference in the binding energies of the orbitals



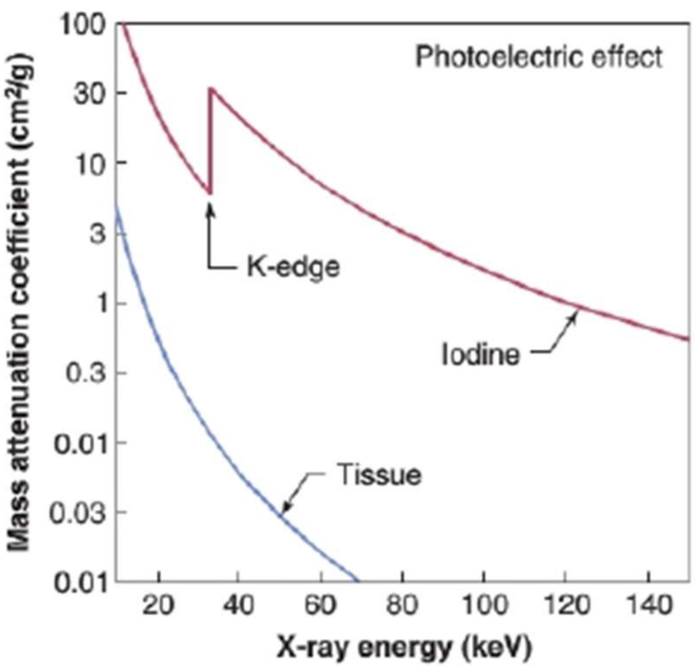
Photoelectric Effect (PE)

$E_\gamma >> E_{BE}$ *(inner shell electron interaction)*

- Probability of photoelectric absorption proportional to Z^3/E^3
- Due to the absorption of the incident x-ray without scatter, *maximum subject contrast arises* with a PE interaction
- Explains why contrast decreases as higher energy x-rays are used in the imaging process (in addition Compton scatter decreasing contrast); as E_o is increased, the probability of PE is decreased eight- fold
- Increased probability of photoelectric absorption just above the inner shell E_{BE} ("absorption edges)

Photoelectric Effect (PE)

$E_{\gamma} >> E_{BE}$ *(inner shell electron interaction)*



- Causes discontinuities in the attenuation profiles
- K-edges become significant factors for higher Z materials as the E_{BE} are in the diagnostic energy range
- "K-edge" is important in diagnostic imaging

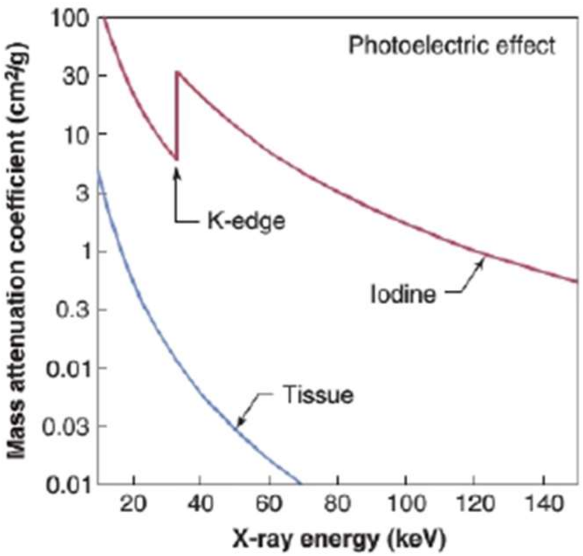
Use in Radiology	Element	Symbo l	Atomi c # (Z)	k-edge Energy
Contrast Agents	Iodine	I	53	33 keV
	Barium	Ba	56	37 keV
	Gadolinium	Gd	64	50 keV
Imaging Detectors	Cesium (DR)	Cs	55	36 keV

Photoelectric Effect (PE)

$$E_\gamma >> E_{BE}$$

(inner shell electron interaction)

- At photon E << 50 keV, the PE plays an important role in imaging soft tissue, amplifying small differences in tissues of slightly different Z and improving subject contrast (e.g., in mammography)



Increased absorption probabilities improve subject contrast and quantum detective efficiency

PE is also the dominant interaction contributing to dose to the patient

Use in Radiology	Element	Symbol	Atomic # (Z)	k-edge Energy
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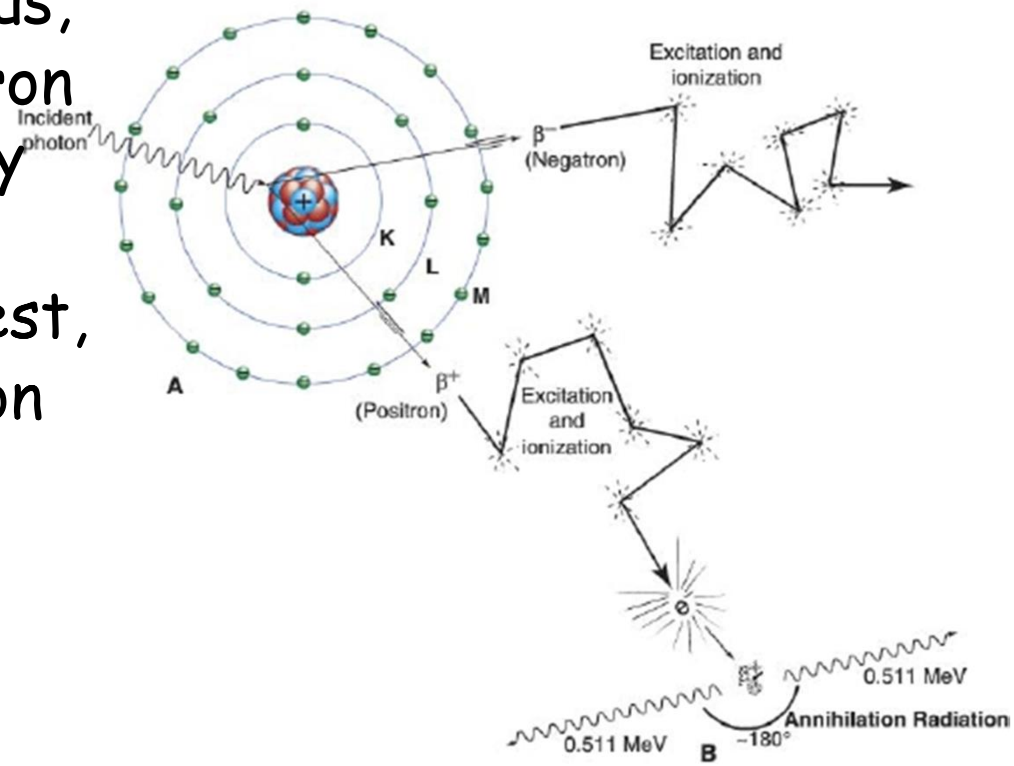
Imaging Detectors	Cesium (DR)	Cs	55	36 keV

Pair Production

$E_\gamma > 1.02\ MeV$

- Only occurs if incident photon has energy exceeding 1.02 MeV (beyond diagnostic & NM energy range)
- Interaction with the nucleus, creating an electron-positron pair (both lose their energy via excitation/ionization)
- When positron comes to rest, it interacts with an electron and produces two 511-keV photons.



X-ray Interactions with Matter (Summary)

Photon interactions

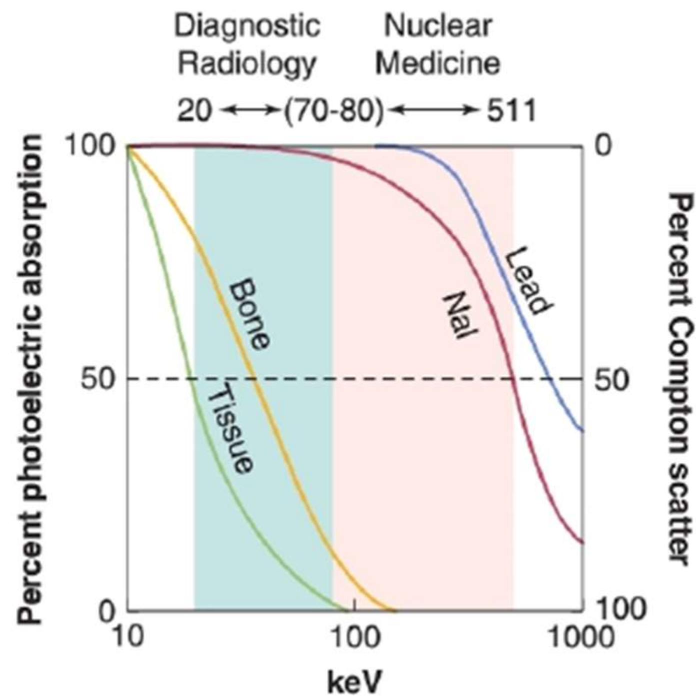
- Rayleigh
- Compton (dominant interaction, primary contribution of scattered photons (for a wide-range of Eo)
- Photoelectric effect (PE, improved contrast b/c of k-Edges, higher probability at low Eo)
- Pair Production (n/a in radiography, used in PET imaging)

Effect on image

Rayleigh Scattering: image degradation (~5% at 70 keV)

Compton Scattering: image degradation

Photoelectric Effect: increased contrast/dose



QUANTIFYING INTERACTIONS

Quantifying Interactions (attenuation)

- What is 'attenuation'?

  - A photon can interact or just pass through the patient's tissue
  - Attenuation

    1. Absorption – Photoelectric
    2. Scattering – Rayleigh, Compton

- How can we estimate how much attenuation is occurring?

  Interactions are a stochastic process, driven by probabilities The factor that attempts to get a handle on this is the *linear attenuation coefficient* (μ)

- fraction of incident photons that will be attenuated per unit thickness
- Units of cm^{-1}
- Must know photon energy and the medium in which the interaction is occurring
- Example: $\mu_{100keV,\ tissue} = 0.16\ cm^{-1}$

N remaining photons

N_o – starting photons

μ – linear atten coeff

x – thickness of material

Attenuation of X-rays and Gamma-Rays

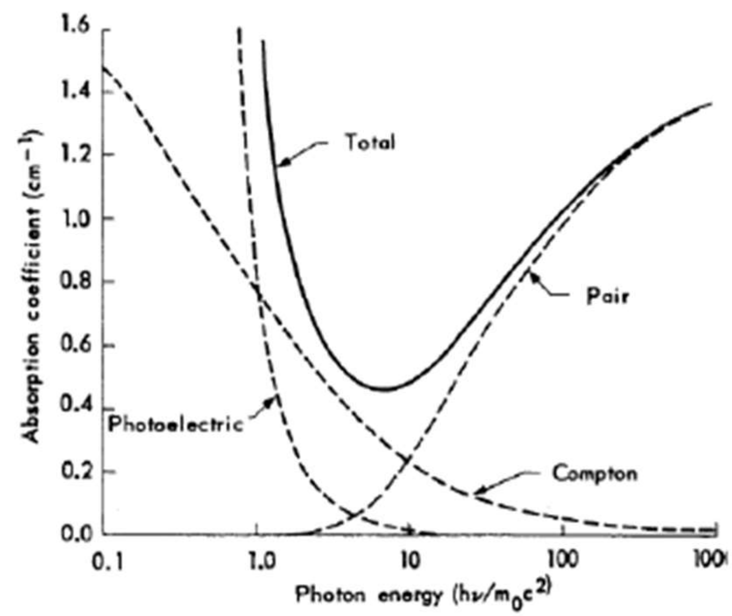
- Fraction of photons removed (due to absorption or scattering) from mono-energetic beam of photons per unit thickness of material is called linear attenuation coefficient :

$$\Delta N = -\mu \times N \times \Delta x\,; \quad \mu = n \times \sigma$$

n: number of atoms per volume ($\rho/A\, N_A$)
σ: cross-section per atom or atomic cross-section ("area")

$$\Rightarrow N(x) = N_0 e^{-\mu x}$$

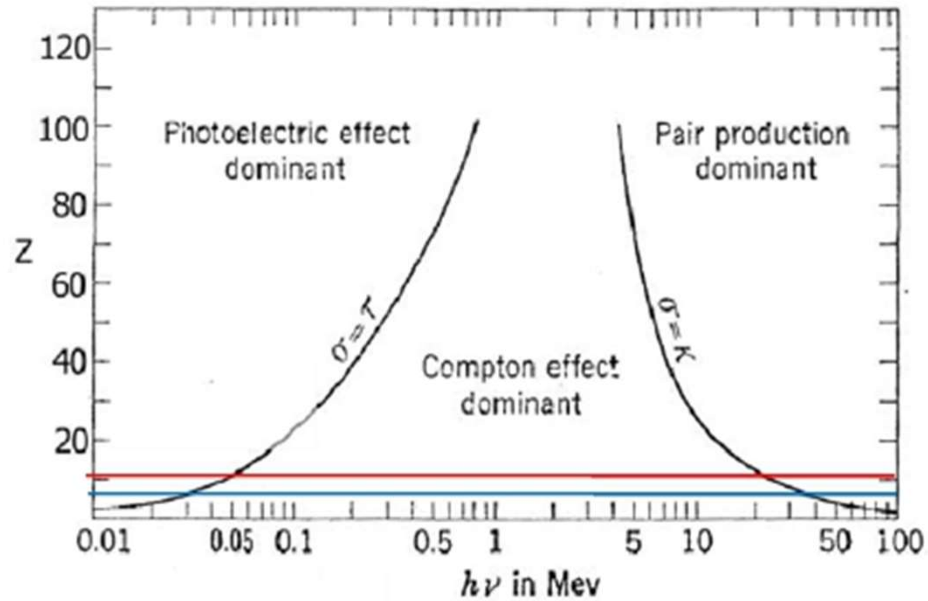
$$\mu_{total} = \mu_R + \mu_{PE} + \mu_{CS} + \mu_{PP}$$



- μ depends on
  - E, Z, A, ρ, ρ_e

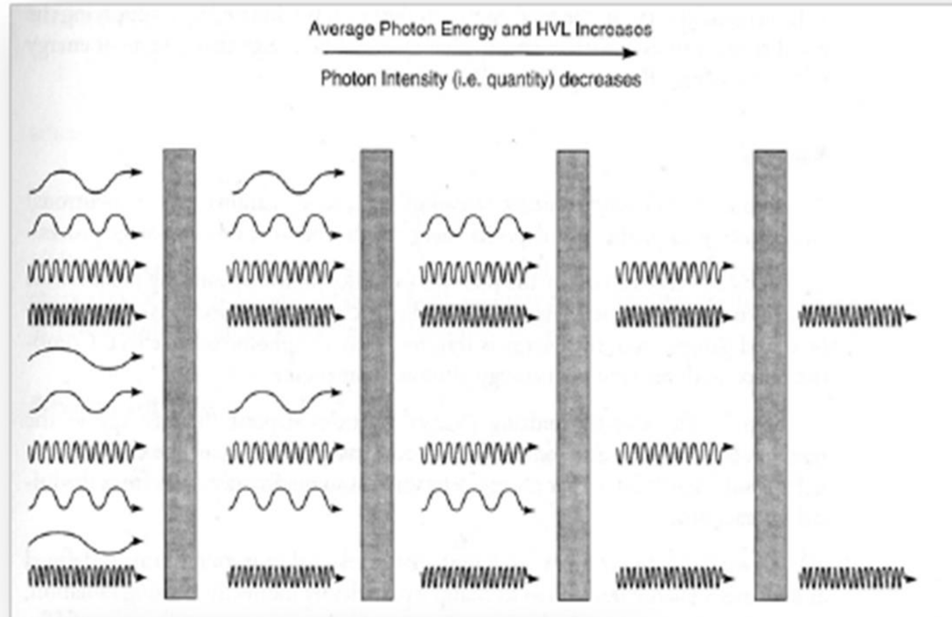
Bone: Z~13
E(C)=35 keV

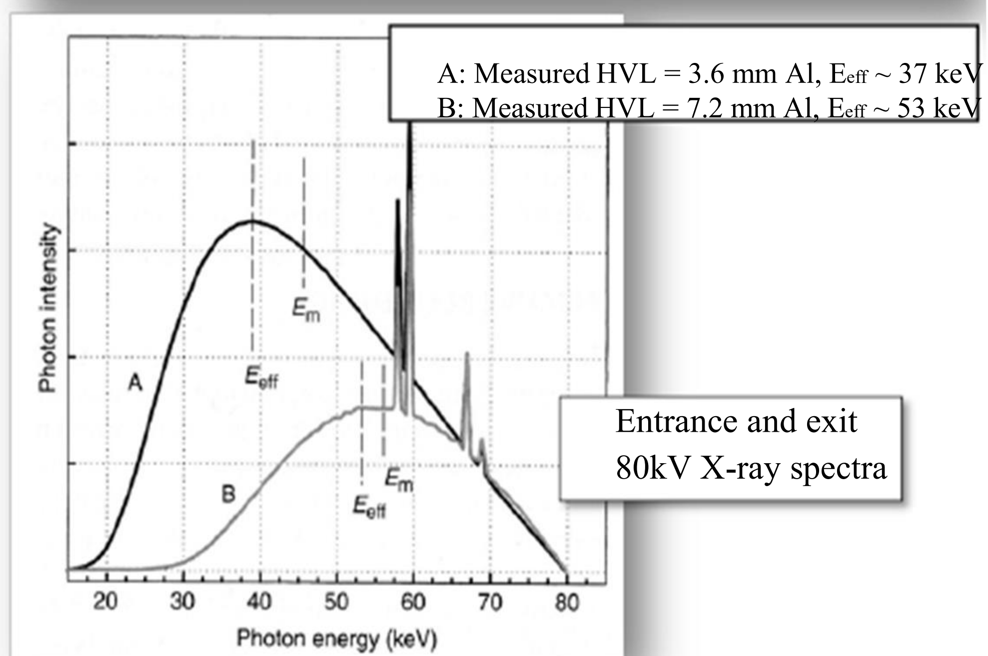
Tissue: Z~7
E(C)=25 keV



Poly-Energetic Beams – Beam Hardening

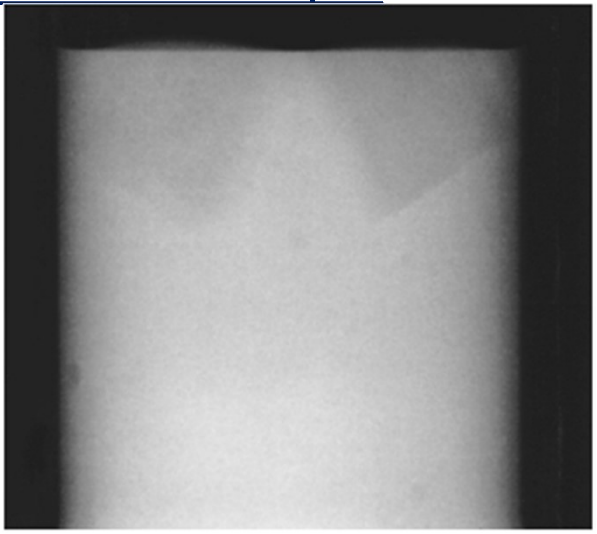
- Poly-energetic photon beam loses smaller energies more quickly than larger energies in a medium.
- Although overall reduction in the number of photons, the average or effective energy increases.
- Low-energy (soft) X-rays will not penetrate most tissues in the body, increasing the dose to the patient and not contributing to the diagnostic quality. Therefore, filters made out of thin plates of e.g. aluminum are used to remove soft X-rays.





For 100 photons @ 100 keV passing through 6-cm of tissue, how many remain?

- $N=N_0e^{-\mu x}=100e^{(-.16x6)}=38\ \%$
- Stated differently, 62% of the original photons were attenuated. Higher energy photons have lower μ and experience lower attenuation. Higher μ indicates higher attenuation and lower transmission
- $\mu_{total} = \mu_{Rayliegh} + \mu_{PE} + \mu_{Comp} + \mu_{PP}$
- The equation above seems to indicate that thickness is the only driver of attenuation
- If we took a radiograph of a cup of ice water, would we be able to see the ice?
- <u>Interaction probability is really proportional to number of atoms per volume (density)</u>



Quantifying Interactions

- Mass attenuation coefficient (μ/ρ) – normalization of the linear attenuation coefficient (μ) for standard unit density (ρ)

- *Example: water, ice, and vapor*

$$\mu_{water} > \mu_{ice} >> \mu_{vapor}$$

$$\mu/\rho_{water} = \mu/\rho_{ice} = \mu/\rho_{vapor}$$

(

Attenuation - Subject Contrast

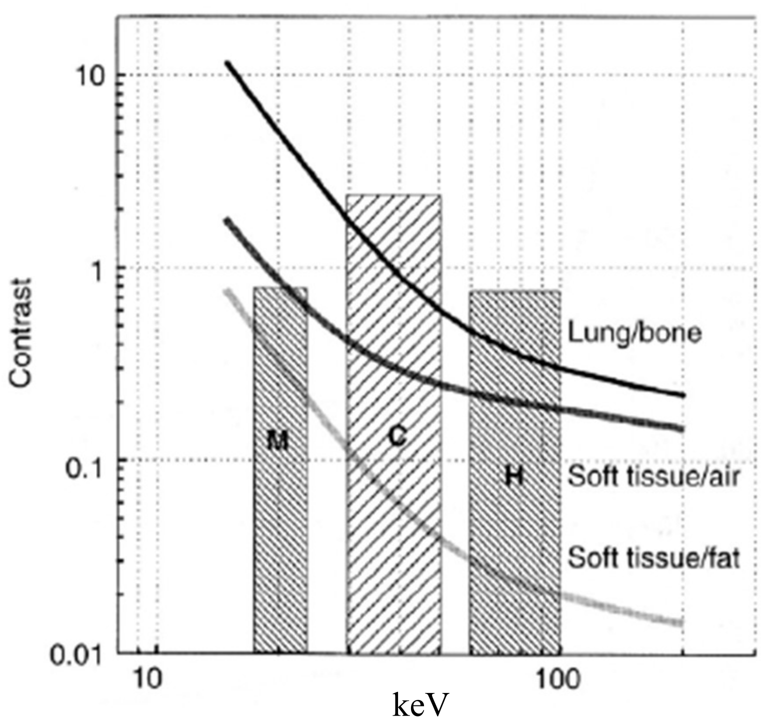
- Transmitted intensities:

$$I_2 = I_0 \times e^{-(\mu_2 x_2)} \times e^{-\mu_1 (x_1 - x_2)}$$

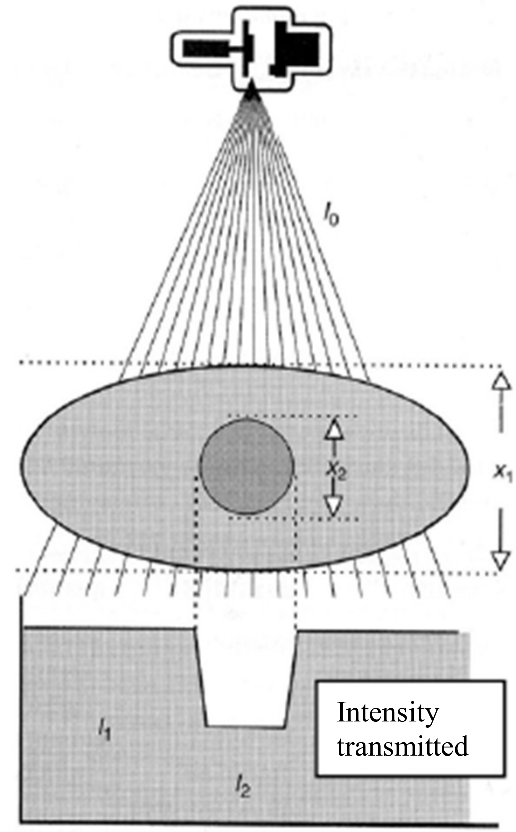
$$I_1 = I_0 \times e^{-(\mu_1 x_1)}$$

- Transmitted fraction/ subject contrast:

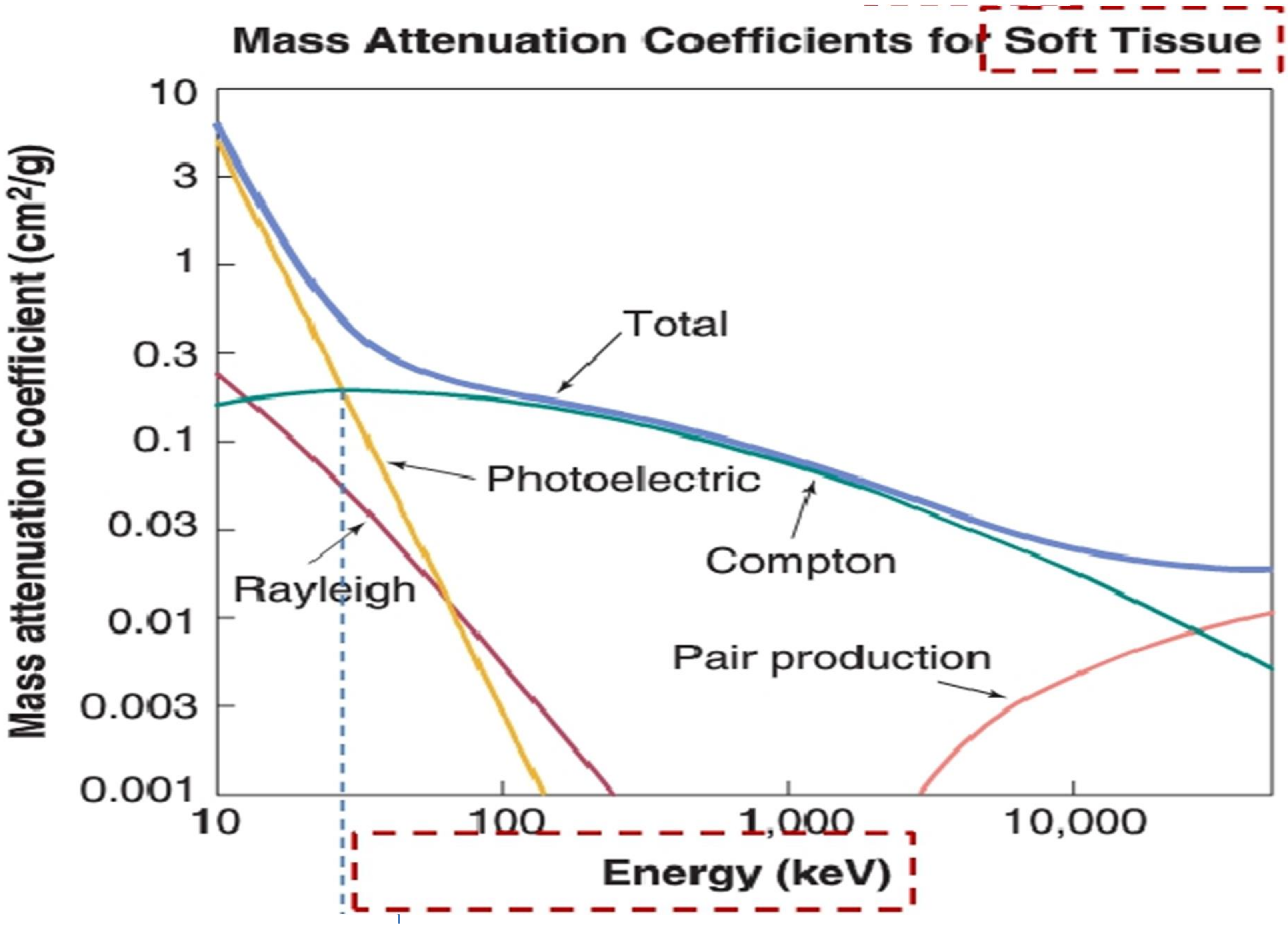
$$I_2 / I_1 = e^{-(\mu_2 - \mu_1) x_2}$$



Subject contrast, i.e. differences in attenuation coefficients between lung and bone, soft tissue and air, and soft tissue and fat.



Effective kilovoltage ranges:
M: Mammography
C: Conventional Radiography
H: High kV investigation (e.g. chest)



Note

Crossover point where dominant interaction transitions between being PE and Compton is ~25 keV in tissue

Interactions – Big Picture

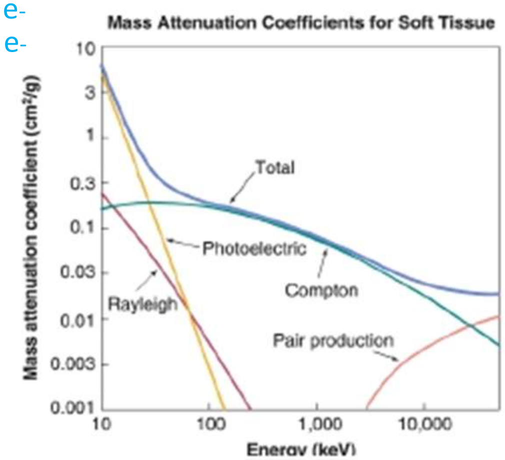
"INTERACTIONS"

- **Particle**
  - **Charged**
    - **Heavy**
      - **α^{2+} - Alpha Particle**
      - **p^{+} - Proton**
        - High LET
        - Path = Range
    - **Light**
      - **e^{-} - Electron**
      - **β^{+} - Positron**
        - Low LET
        - Path > Range
  - Un-Charged
    - n^{o} - Neutron
      - Fast
      - Intermediate
      - Thermal
- **Photon**
  - **Gamma Ray** – Originate from nucleus
  - **X-ray** – Originate from electron interactions/states
    - Rayleigh Scattering
    - **Compton Scattering**
      - IN: photon + target outer shell e-
      - OUT: scattered photon + ejected e-
      - Energy shared b/w photon/e-
      - Weak Z-dependence @ $E_{diagnostic}$
    - **Photoelectric Effect**
      - IN: photon + target inner shell e-
      - OUT: ejected e- (photoelectron)
      - OUT: characteristic x-ray or Auger e-
      - Z^3/E^3 dependence
    - Pair Production
      - Incident E_{photon} must be > B_{elec}

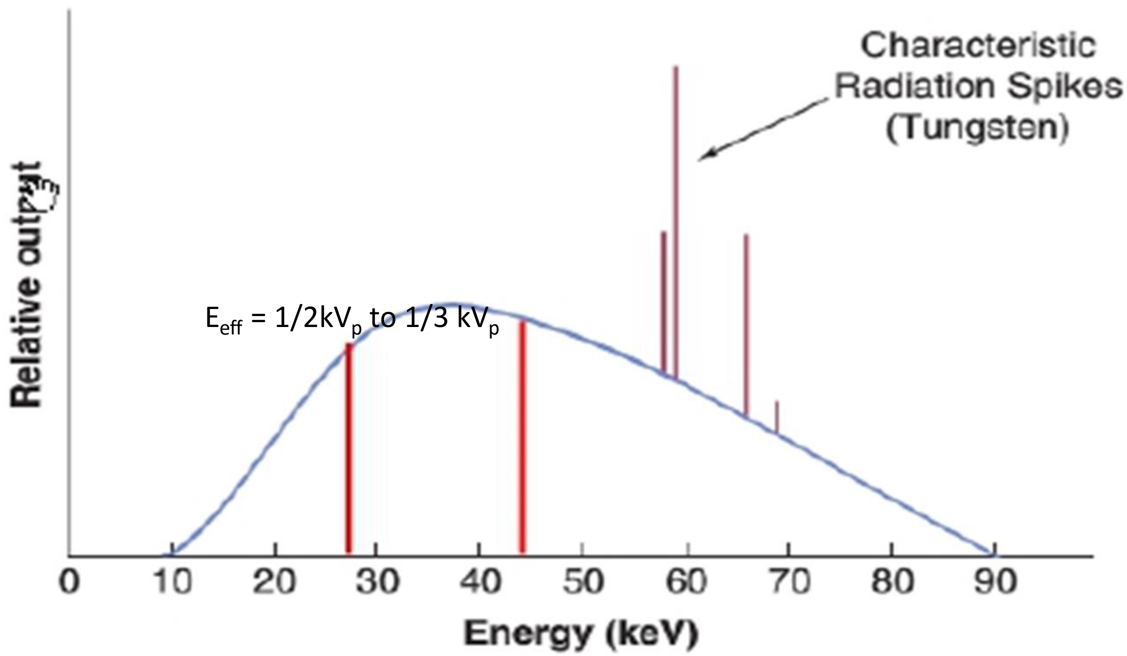
Coulombic Interactions

A) Collisional Losses
- Excitation
- Ionization

B) Radiative Losses
- Bremmstrahlung



- x-ray spectra, have a range of energies and every x-ray tube can be different

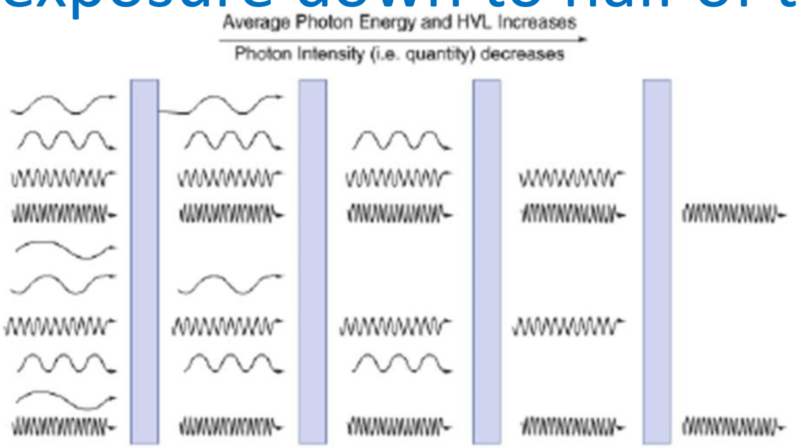


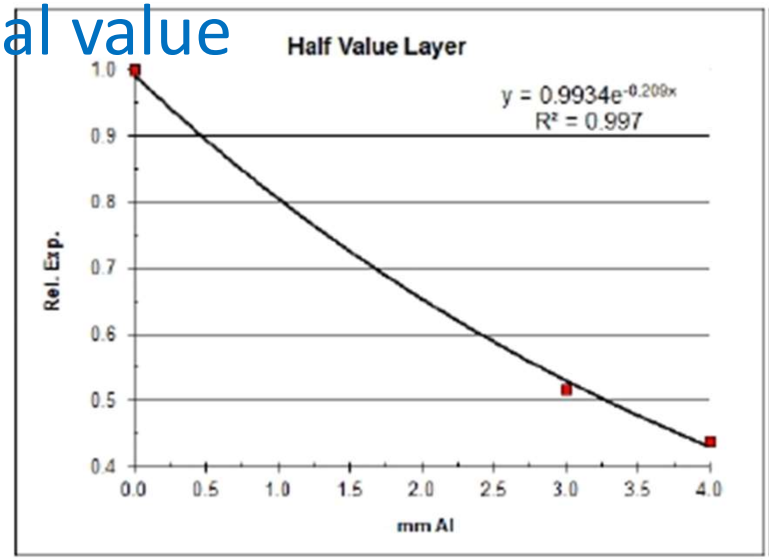
- PROBLEM: Difficult to measure (especially accurately)
- PRACTICALITY: Half Value Layer (HVL) Measurement

Quantifying Interactions

Half Value Layer

- Half Value Layer –
  The thickness of Aluminum needed to reduce the exposure down to half of the initial value





- This takes advantage of the fact that photons of different energies have different penetrabilities

From same tube, 80 kVp or 100 kVp has higher HVL?
Two different tubes, 80 kVp in both cases, one has higher HVL. Why?

Photon and Energy Flux and Fluence
- Definitions -

- Fluence: $$\Phi = \frac{Photons}{Area} \quad \left[\frac{1}{cm^2}\right]$$

- Flux: $$\phi = \dot{\Phi} = \frac{Photons}{Area \times time} \quad \left[\frac{1}{cm^2 \times s}\right]$$

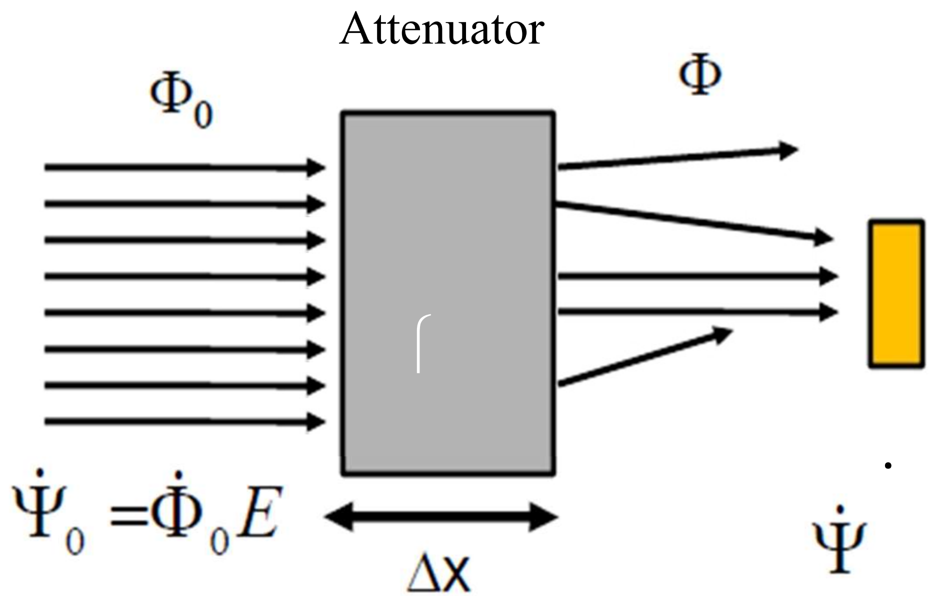
- Energy fluence: $$\Psi = \frac{Photons}{Area} \times \frac{Energy}{Photon} = \Phi \times E \quad \left[\frac{keV}{cm^2}\right]$$

- Energy flux: $$\psi = \dot{\Psi} = \frac{Photons}{Area \times time} \times \frac{Energy}{Photon} \quad \left[\frac{keV}{cm^2 \times s}\right]$$

(or intensity in [J/m2s])

Energy Transfer and Energy Absorption

- Assume uniform, broad, parallel beam of mono-energetic photons normally incident on an absorber of thickness ⊗x
- Detector receives uncollided as well as scattered and other (e.g. Bremsstrahlung and fluorescence) photons
- Thus, not all of the energy of the interacting photons are necessarily absorbed
- The decrease in beam intensity with increasing x can be expected to be less than described by the linear attenuation exp(-⌠x)
- The energy transfer coefficient describes the actual energy that was transferred (to the short-range electrons) in the interaction process



Fluence: Φ [photon/cm^2]
Incident energy fluence rate: $\dot{\Psi}_0$
Transmitted energy fluence rate: $\dot{\Psi}$
Linear Attenuation Coefficient: μ [1/cm]

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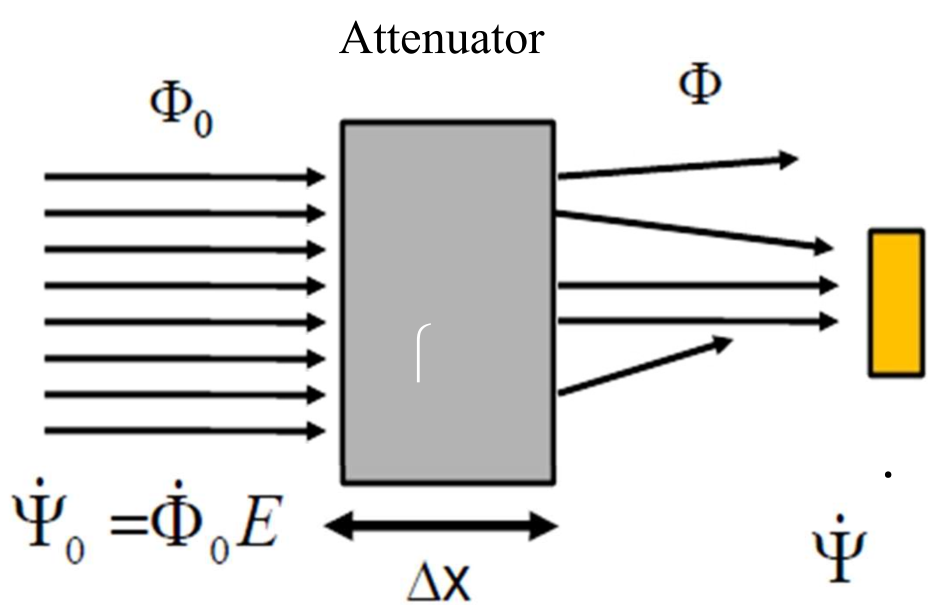

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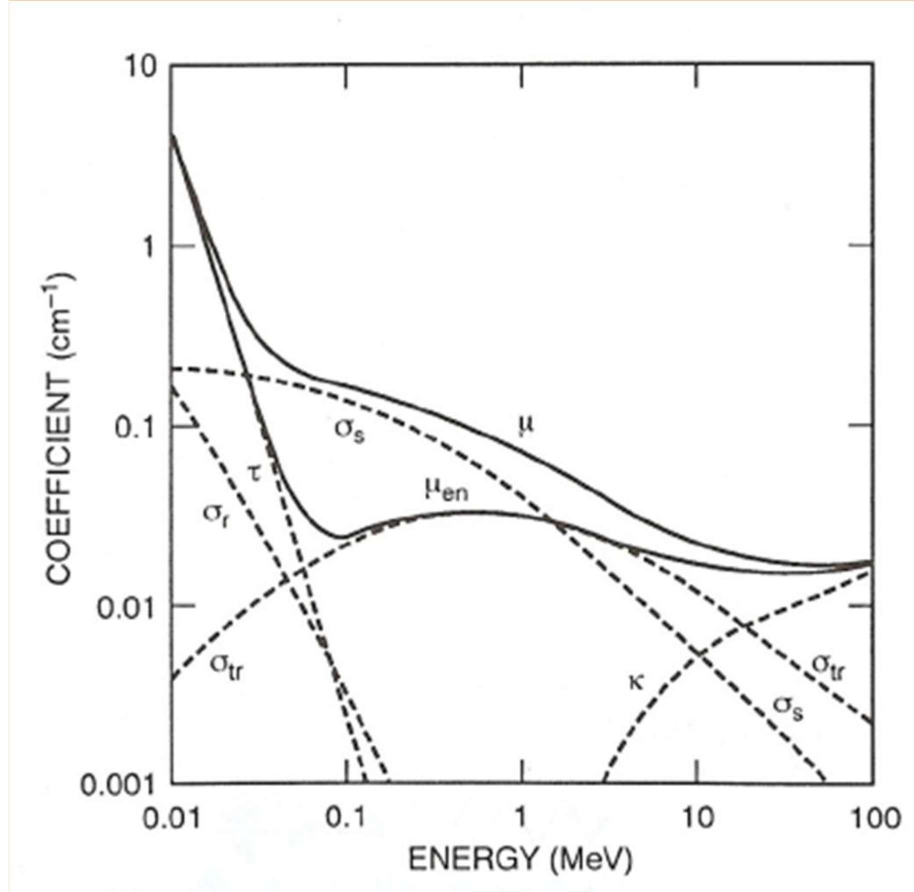
Transmitted energy fluence rate: $\dot{\Psi}$
Linear Attenuation Coefficient: μ [1/cm]

Mass Energy Transfer & Energy Absorption Coefficients

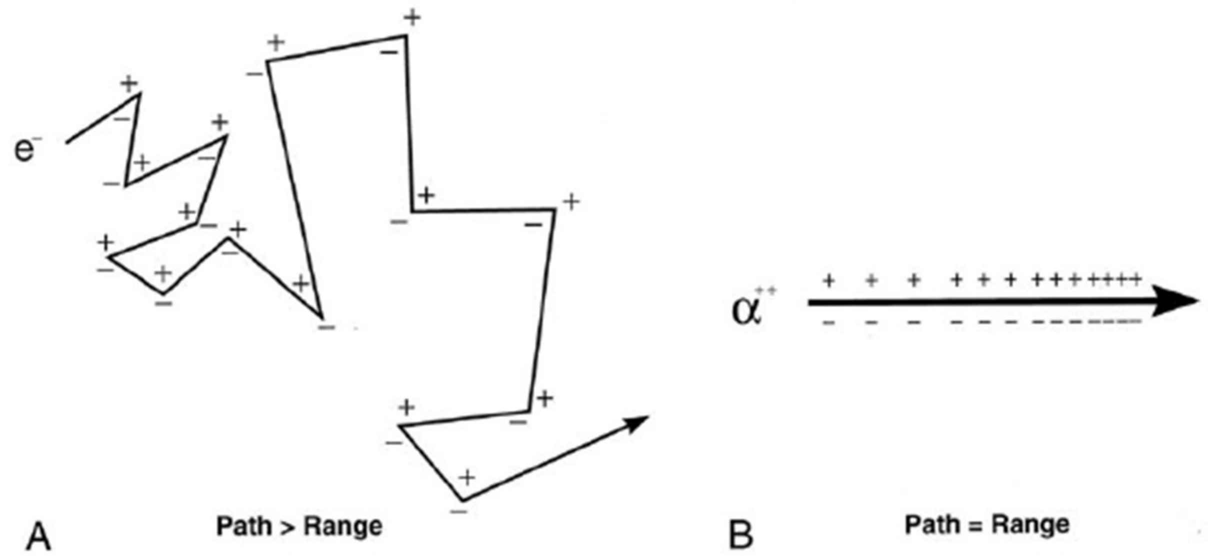
Photon Energy (MeV)	Water			Lead		
	μ/ρ	μ_{tr}/ρ	μ_{en}/ρ	μ/ρ	μ_{tr}/ρ	μ_{en}/ρ
0.01	5.33	4.95	4.95	131.	126.	126.
0.10	0.171	0.0255	0.0255	5.55	2.16	2.16
1.0	0.0708	0.0311	0.0310	0.0710	0.0389	0.0379
10.0	0.0222	0.0163	0.0157	0.0497	0.0418	0.0325
100.0	0.0173	0.0167	0.0122	0.0931	0.0918	0.0323



Quantifying Interactions

Linear Energy Transfer

- Another thing to keep in mind, different particles deposit energy in different ways.



- Linear energy transfer (LET) – amount of energy deposited per unit length (eV per cm)
  - High LET more damaging than low LET radiation